ANALYSIS OF TENSEGRITY-BASED PARALLEL PLATFORM DEVICES

By

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A THESIS PRESENTED TO THE GRADUATE SCHOOL
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A parallel-platform device that is based on tensegrity is studied in this paper. A tensegrity structure is one that is tensionally continuous and compressionally discontinuous. It comprises several noncompliant struts, which are in compression along with ties, both elastic and inelastic, which are in tension. The device studied in this thesis replaces the noncompliant members of a tensegrity structure with prismatic actuators, and each elastic member with a cable-spring combination in series. The length of the cable is adjustable. These changes are made so that the device can have six degrees of freedom, i.e. the length of the three prismatic actuators and the length of the three cables that are in series with the springs. The study performed in this thesis shows that the external wrench can act along any screw that is independent of five of the device legs. This study also shows that the device's compliance characteristics can be varied while maintaining its position and orientation. A reverse analysis of the device's position and orientation, along with its internal potential energy, is performed. The effect of a seventh leg, another prismatic actuator, is also analyzed and found to satisfactorily implement the needed external wrench. The position of the seventh leg needs to be variable in order for it and five other legs to remain linearly independent.

CHAPTER1
INTRODUCTION

A 3-3 in-parallel platform device consists of two rigid platforms connected by six noncompliant legs. The legs are connected to the platforms with ball-and-socket joints. The length of each leg is variable. If the bottom platform is connected to ground, then the top platform retains six degrees of freedom; the freedoms to translate along the x, y, and z-axes, and to rotate about each of them. A 3-3 in-parallel platform is pictured below in Figure 1-1.

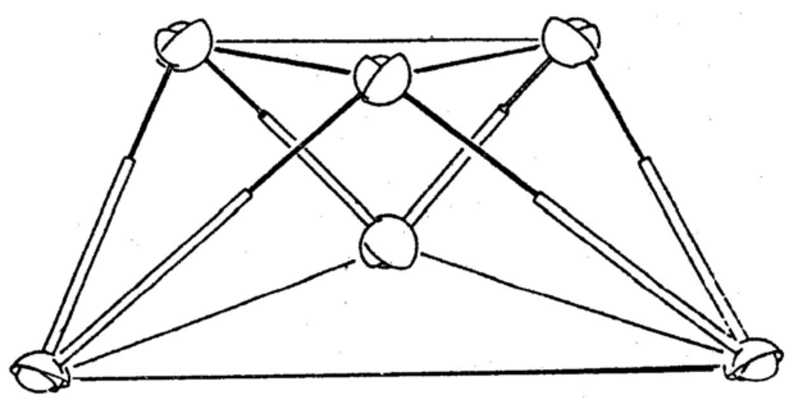

Figure 1-1. 3-3 Parallel Platform Device

A device of this type is immobile for a given set of leg lengths. If the legs were elastic then an external force could displace the platform. Its harmonic frequency would also be affected by altering the elasticity of its legs. A tensegrity structure is a loose example of a 3-3 in-parallel platform with fixed leg lengths, some of the legs being elastic.

Buckminster Fuller assigned a meaning to the word tensegrity. The word refers to the phenomena that all objects in the universe exert a pull on each other and thus the universe is tensionally continuous. The universe is also compressionally discontinuous [4]. A tensegrity structure is an illustration of this phenomenon on a man-sized scale.

A tensegrity structure is pictured in Figure 1-2, below. This structure is a triangular tensegrity prism. A tensegrity structure consists of multiple members, some of which are solely in tension; the others are in compression. None of the compressional members come in contact with another compressional member. However, the members in tension are different in this respect.

Any vertex of the structure can be connected to another point on the structure by tracing a line along tensional members. This is evidence of the structure's tensional continuity. The members labeled "S" in Figure 1-2 are compressional. The remaining members are ties, and thus can only be in tension. This structure, without the application of an external force, can only be in equilibrium at two positions where one is simply a mirror image of the other reflected through the base plane. A condition on this position will be discussed in the next chapter.

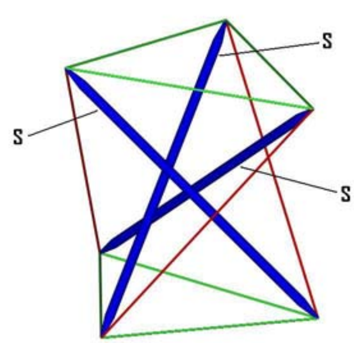

Figure 1-2. Tensegrity Structure

A parallel prism is the root of a tensegrity prism. The parallel prism has ties that are parallel and have fixed lengths. A tensegrity prism is created by taking a parallel prism and rotating the top plane about its central, perpendicular axis by an angle α, and then inserting non-compliant members connecting the erstwhile end-points of the diagonals of the planes made of the parallel ties [6]. For a triangular parallel prism there are three such planes, thus there are three non-compliant members in a triangular tensegrity prism.

Tensegrity prisms, like parallel prisms, can be of any degree polygon. Kenner [5] found the rotation angle, α, for the general tensegrity prism as

$$\alpha = \frac{\pi}{2} - \frac{\pi}{n} \tag{1.1}$$

where n is the number of sides in the polygon of the upper or lower plane.

A different property of tensegrity prisms is calculated to be independent of the value of n. Knight et al. [6] show that, due to the arrangement of its members; a triangular tensegrity structure has instantaneous mobility. This is a characteristic of all tensegrity prisms. Tensegrity prisms share another interesting characteristic; they are deployable from a bundled position.

Duffy et al. [3] analyze the deployable characteristics of elastic tensegrity prisms, as do Tibert [11] and Stern [10]. Tensegrity prisms have this attribute because the potential energy stored in their elastic members is greater when bundled than when in the position of Figure 1-2. Indeed, this figure illustrates a position of minimum potential energy. The allure of tensegrity structures encompasses more than their characteristics of motion.

Burkhart [2] fabricates and analyzes domes using triangular tensegrity prisms. The triangular tensegrity prisms that he uses have smaller tops than bottoms. Others have studied the uses of tensegrity prisms when the top and bottom are the same size, but the lengths of the side ties are variable.

Tran [12] discusses a device comprising three compliant ties and three non-compliant struts, all of which have adjustable lengths. Each compliant leg consists of a non-elastic cable in parallel with an elastic member, a spring. These legs can be called side ties. This device is a triangular tensegrity prism with variable lengths. Oppenheim [8] also deals with adjustable-member-length tensegrity prisms. The aim of these devices is to allow a tensegrity structure to achieve varied positions.

These devices can have different stiffness values for identical postures because of the variable nature of the compliant member lengths. Skelton [9] discusses using tensegrity structures with variable member lengths in wing construction. Doing this would allow the wing to have adjustable stiffness or shape. The thesis presented here evaluates a method of calculating the conditions necessary to attain variable configurations of a triangular-tensegrity-prism based device.

Tran states that because the ties of its top and bottom are non-compliant, the device is effectively a parallel mechanism. Lee [7] suggests a parallel mechanism in which the joints are offset along the platform sides. A parallel mechanism with three compliant legs and offset joints is pictured in Figure 1-3. This mechanism serves as the model for analysis in this paper.

The top and bottom ties of a tensegrity structure are, in this device, replaced with platforms. If three ties that form a triangle are kept in tension with forces acting only at the corners of the triangle, then the shape of the triangle cannot change. This is true regardless of any change in the positions of the corners relative to each other. It follows that as long as the struts are in compression and the side ties in tension, and the struts and side ties meet at only three points per platform, then a platform would behave the same as the three ties found in the top and bottom of the tensegrity structure in Figure 1-2. The device in Figure 1-3, however, does not meet all the above criteria.

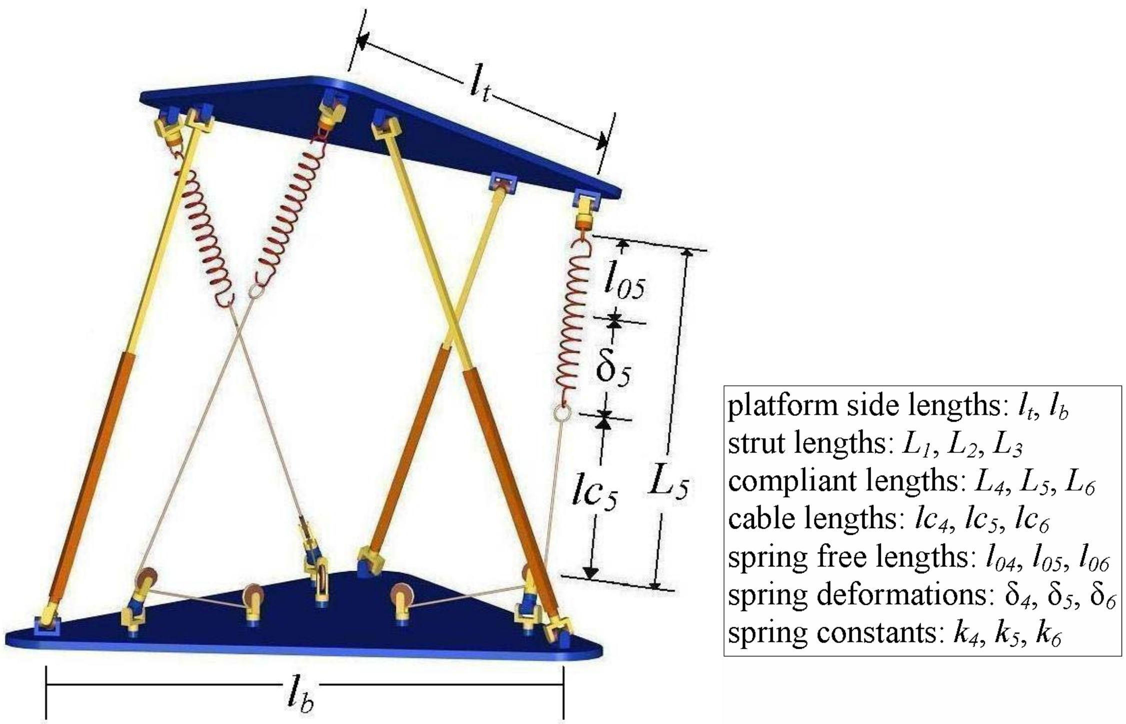

Figure 1-3. Tensegrity-Based Parallel Platform Device

The joints in Figure 1-3 are not connected at only three points per platform. Since the device is modeled after the special 3-3 Parallel Platform studied by Lee, the joints are separated along one side of the triangle. This is done to increase the quality index of the robot at home position [13]. For the analysis in this paper, the placement of the joints on the platforms will not hinder the acquisition of results.

Problem Statement. The given information in the problem is listed here:

- The lengths of the sides of the top and bottom platforms of the device (l_t, l_b)
- The position and orientation of the top platform relative to the bottom, or the transformation matrix (T)
- The spring free lengths (l_{01}, l_{02}, l_{03})
- The spring constants (k_1, k_2, k_3)
- The potential energy stored in the springs (U)
- The screw along which an external wrench acts ($\$_{ext}$)

The need for an external wrench will be discussed in chapter two, section eight.

The values sought are listed below:

- The length of each strut (L_1, L_2, L_3)
- The length of cable in each compliant leg (l_{c4}, l_{c5}, l_{c6})
- The spring deformations (δ_4, δ_5, δ_6)

For a reverse analysis of a parallel-platform device, it is desired to find the six leg lengths for a given position and orientation. In order to position a device like this, the length of the cable portion of each compliant leg would be required. Thus the knowledge of the total length of each compliant leg is useless without the value of the spring deformation for each leg. The spring deformations are accordingly part of the desired information.

CHAPTER 2
REVERSE ANALYSIS SOLUTION FOR TENSEGRITY-BASED 3-3 PARALLEL PLATFORMS

This chapter will present a solution for the leg lengths of a tensegrity-based 3-3 parallel platform device when given its position and orientation. This is also known as a reverse analysis solution. The reverse analysis presented here will differ from that of the non-tensegrity-based device because it deals with three compliant legs and because the potential energy in these members is part of the information given in the problem statement.

2.1 Defining the Vertices in a Local Coordinate System

There are either three or six vertices on each platform, depending on if the joints are separated along the sides of the platforms as in Figure 1-3 or not. The amount of separation is denoted by the variable σ. This variable represents the portion of the platform side, by which the joints are separated. The equations of the six joint coordinates exist in pairs. The joints are labeled in Figure 2-1.

The local coordinates of the first joint in each pair are found by setting σ equal to zero. The coordinates of the second joint in each pair are found by using the σ given in the platform geometry. The equations for the locations of the platforms' joint pairs are

$$^{B}\mathbf{P}_1 = {}^{T}\mathbf{P}_4 = \begin{bmatrix}0 & 0 & 0\end{bmatrix}, \tag{2.1a}$$

$$^{B}\mathbf{P}_{61} = {}^{T}\mathbf{P}_{14} = \begin{bmatrix}\sigma\cdot\cos\left(\frac{\pi}{3}\right) & \sigma\cdot\sin\left(\frac{\pi}{3}\right) & 0\end{bmatrix}, \tag{2.1b}$$

$$^{B}\mathbf{P}_2 = {}^{T}\mathbf{P}_5 = \begin{bmatrix}l_0 & 0 & 0\end{bmatrix}, \tag{2.2a}$$

$$^{B}\mathbf{P}_{42} = {}^{T}\mathbf{P}_{25} = \left[(1-\sigma)\cdot l_0 \quad 0 \quad 0\right], \tag{2.2b}$$

$$^{B}\mathbf{P}_{3} = {}^{T}\mathbf{P}_{6} = \left[\frac{l_0}{2} \quad l_0 \sin\left(\frac{\pi}{3}\right) \quad 0\right], \tag{2.3a}$$

$$^{B}\mathbf{P}_{53} = {}^{T}\mathbf{P}_{36} = \left[(1+\sigma)\frac{l_0}{2} \quad (1-\sigma)\cdot l_0 \sin\left(\frac{\pi}{3}\right) \quad 0\right], \tag{2.3b}$$

The different superscripts in each of Equations (2.1) – (2.3) refer to the fact that these point coordinates are measured in local systems. Thus the origin of the bottom and top coordinate systems are at points $\mathbf{P}_1$ and $\mathbf{P}_4$, respectively. Their x-axes run along the lines from $\mathbf{P}_1$ to $\mathbf{P}_2$ and from $\mathbf{P}_4$ to $\mathbf{P}_5$. The z-axes of the coordinate systems are perpendicular to the platforms. From these equations, and Figure 2-1, it is apparent that with σ equal to zero, $\mathbf{P}_1$ would be coincident with $\mathbf{P}_{14}$ and likewise for all other pairs.

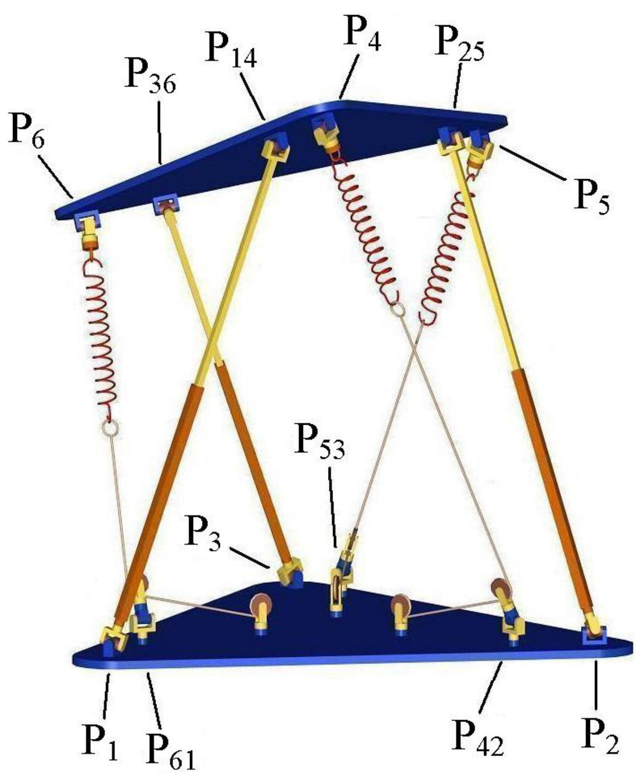

Figure 2-1. Tensegrity Mechanism with Points Labeled

2.2 Coordinate Transformations

Often, the information for position and orientation of the top platform is given as a transformation matrix. This is a four-by-four matrix. It provides the values needed to describe the relative position and orientation of two coordinate systems. One way to define the transformation matrix is to multiply four specific matrices in a specific order. For example, the top coordinate system can be thought of as initially aligned with the bottom coordinate system. It is then translated to some point whose coordinates can be written as [x, y, z]. Then it is rotated about the current x axis by α, followed by a rotation of β about the modified y axis, and finally followed by a rotation of γ about the modified z axis. For this example, the transformation matrix that relates the top and bottom coordinate systems, ${}^{B}_{T}\mathbf{T}$, can be calculated

$$
{}^{B}_{T}\mathbf{T}=\begin{bmatrix}1&0&0&x\\0&1&0&y\\0&0&1&z\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&\cos(\alpha)&-\sin(\alpha)&0\\0&\sin(\alpha)&\cos(\alpha)&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos(\beta)&0&\sin(\beta)&0\\0&1&0&0\\-\sin(\beta)&0&\cos(\beta)&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos(\gamma)&-\sin(\gamma)&0&0\\\sin(\gamma)&\cos(\gamma)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}. \quad (2.4)
$$

The coordinates of any point that is known in the top coordinate system can be determined in the bottom coordinate system as

$$
{}^{B}\mathbf{P}_1={}^{B}_{T}\mathbf{T}\;{}^{T}\mathbf{P}_1 \quad (2.5)
$$

where ${}^{T}P_1$ represents the coordinates of point 1 written in homogeneous coordinates in terms of the top coordinate system as

$$
{}^{T}\mathbf{P}_1=\begin{bmatrix}x_1\\y_1\\z_1\\1\end{bmatrix} \quad (2.6)
$$

and similarly, ${}^{B}P_1$ represents the coordinates of point 1 written in homogeneous coordinates with respect to the bottom coordinate system.

2.3 Plücker Line Coordinates

Plücker coordinates are a way to describe a line in space. The term $\mathbf{\$}_L$ will be used to represent the coordinate of a line which consists of six constants arranged in a column matrix as

$$\mathbf{\$}_L = \begin{bmatrix} l \\ m \\ n \\ p \\ q \\ r \end{bmatrix} . \quad (2.7)$$

The variables *l*, *m*, and *n* represent respectively the x, y, and z directions of the line, while *p*, *q*, and *r* signify the moments of the line about the x, y, and z-axes respectively. These moment terms can be calculated as the cross product of the coordinates of a point on the line with the direction of the line. In this thesis it is required to know the coordinates of the lines representing the mechanism legs. These lines can be seen as a group of lines, each of which are defined by two points in space.

Plücker line coordinates can be used to describe the line connecting two points, $\mathbf{P}_1$ and $\mathbf{P}_2$, which exist in space. The equations for the Plücker coordinates of this line are written as

$$l = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \end{vmatrix}, \quad m = \begin{vmatrix} 1 & y_1 \\ 1 & y_2 \end{vmatrix}, \quad n = \begin{vmatrix} 1 & z_1 \\ 1 & z_2 \end{vmatrix}$$
$$p = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}, \quad q = \begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}, \quad r = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \quad (2.8)$$

where the term x_1 is the x component of the position vector of point $\mathbf{P}_1$; y_2 is the y component of the position vector of point $\mathbf{P}_2$, etc.

Plücker line coordinates are homogeneous coordinates, i.e. multiplying all six coordinates by a scalar value results in the same line in space. In this application, the Plücker line coordinates will be written such that the direction of the line is a unit vector. This is readily accomplished by dividing all six of the terms by the magnitude of the x, y, and z components of the line as

$$\mathbf{\$}_L = \begin{bmatrix} \frac{l}{\text{magnitude}} \\ \frac{m}{\text{magnitude}} \\ \frac{n}{\text{magnitude}} \\ \frac{p}{\text{magnitude}} \\ \frac{q}{\text{magnitude}} \\ \frac{r}{\text{magnitude}} \end{bmatrix} \tag{2.9}$$

where the value of *magnitude* is given by

$$magnitude = \sqrt{l^2 + m^2 + n^2} \quad . \tag{2.10}$$

2.4 Screw Coordinates and Wrenches

A screw is a line with a pitch where 'h' is used to denote the pitch. It is important to note that the pitch term has units of length. A screw, **\$**, can be written as

$$\mathbf{\$} = \begin{bmatrix} l \\ m \\ n \\ p + l\,h \\ q + m\,h \\ r + n\,h \end{bmatrix} . \tag{2.11}$$

A screw multiplied by an angular velocity magnitude describes the instantaneous motion of one body relative to another, i.e. a rotation about the line of axis of the screw combined with a translation along the screw axis direction. This is called a twist, $\hat{\mathbf{T}}$. Further, a screw multiplied by a force magnitude represents a force and moment acting on a body, i.e. a force along the line of action of the screw combined with a moment about the screw axis direction. This is called a wrench, $\hat{\mathbf{w}}$.

It is shown in Ball [1] that any combination of forces and moments acting on a rigid body can be replaced by a single equivalent wrench. Further it is shown that this equivalent wrench can be determined by simply adding the screw coordinates of the individual forces and moments that are acting on the body. This is one of the elegant aspects of screw theory.

2.5 Obtaining the Unitized Plücker Coordinates for the Legs

The Plücker coordinates of the legs are found by using the point coordinates of the joints in Equation (2.8). The local point coordinates for the joints are found using Equation (2.1), (2.2), and (2.3).

The global point coordinates for the joints of the bottom platform are given by the three preceding equations, substituting l_b for l_0. The coordinates, relative to the origin of the top platform's coordinate system, of the top platform's joints are also given by that equation using l_t instead of l_0. The local coordinates of the top joints must be transformed into coordinates relative to the bottom platform.

The local coordinates of the top joints are substituted into Equation (2.5) to yield their positions relative to the base. The value of ${}^{B}_{T}\mathbf{T}$ is supplied in the problem statement.

The availability of the absolute point coordinates for the joints yields line coordinates for the legs.

The Plücker coordinates for the legs are obtained by using Equation (2.8). For each strut, the corresponding top-platform joint is used as $\mathbf{P}_2$ in that equation, and that strut's joint that connects it to the bottom platform is used for $\mathbf{P}_1$. For the side ties this substitution was reversed. The joint on the top platform for each tie was used as $\mathbf{P}_1$ and that of the bottom for $\mathbf{P}_2$. The reason for doing this is given in the next section. After obtaining the line coordinates by this method, they are still not useful in the description of forces. This is necessary to obtain the spring lengths.

In order to be used in force description the Plücker coordinates of the line along which a force acts must be unitized. The coordinates of the lines coincident with the legs are unitized using Equation (2.9).

2.6 Obtaining Spring Elongation Values

In this thesis, the deformations of the springs are found by using a force balance equation on the top platform combined with the value of potential energy given in the problem statement. Knowledge of the deformed spring lengths will yield the length of cable paid out in each side tie, part of the information sought in the problem statement.

There are seven wrenches acting on the platform. Six result from the legs of the mechanism. The seventh wrench is an externally applied wrench that acts along the screw given in the problem statement. In order for the device to be in equilibrium, the sum of these forces and moments must be equal to zero. The force and moment balance equation for the top platform can be written as

$$f_1\,\mathbf{\$}_{L1} + f_2\,\mathbf{\$}_{L2} + f_3\,\mathbf{\$}_{L3} + f_4\,\mathbf{\$}_{L4} + f_5\,\mathbf{\$}_{L5} + f_6\,\mathbf{\$}_{L6} + f_{ext}\,\mathbf{\$}_{ext} = \mathbf{0}\,. \tag{2.12}$$

The lines $\mathbf{\$}_{L1}$, $\mathbf{\$}_{L2}$, and $\mathbf{\$}_{L3}$ in Equation (2.12) were taken in the direction going from the bottom-platform joint to the top-platform joint of each leg because of the assumption that those legs would be in compression and thus their forces would act toward the top platform. The lines $\mathbf{\$}_{L4}$, $\mathbf{\$}_{L5}$, and $\mathbf{\$}_{L6}$ were taken in the opposite direction because they can only exert tensional forces. These lines coordinates were then unitized and placed in Equation (2.12).

Equation 2.6.1 can be rearranged as

$$f_1\,\mathbf{\$}_{L1} + f_2\,\mathbf{\$}_{L2} + f_3\,\mathbf{\$}_{L3} + f_5\,\mathbf{\$}_{L5} + f_6\,\mathbf{\$}_{L6} + f_{ext}\,\mathbf{\$}_{ext} = -\,f_4\,\mathbf{\$}_{L4} \tag{2.13}$$

and this equation can be written in matrix format as

$$\begin{bmatrix}\mathbf{\$}_{L1} & \mathbf{\$}_{L2} & \mathbf{\$}_{L3} & \mathbf{\$}_{L5} & \mathbf{\$}_{L6} & \mathbf{\$}_{ext}\end{bmatrix}\begin{bmatrix}f_1\\ f_2\\ f_3\\ f_5\\ f_6\\ f_{ext}\end{bmatrix} = -f_4\,\mathbf{\$}_{L4} \tag{2.14}$$

where the term $[\mathbf{\$}_{L1}\ \ \mathbf{\$}_{L2} \ldots \mathbf{\$}_{ext}]$ is a 6×6 matrix whose columns are the unitized coordinates of the lines along legs 1, 2, 3, 5, and 6 and the unitized screw coordinates of the given external wrench.

Both sides are then divided by the force magnitude f_4. The result can be written as

$$\begin{bmatrix}\mathbf{\$}_{L1} & \mathbf{\$}_{L2} & \mathbf{\$}_{L3} & \mathbf{\$}_{L5} & \mathbf{\$}_{L6} & \mathbf{\$}_{Lext}\end{bmatrix}\begin{bmatrix}f_1'\\ f_2'\\ f_3'\\ f_5'\\ f_6'\\ f_{ext}'\end{bmatrix} = -\mathbf{\$}_{L4} \tag{2.15}$$

where

$$f_n' = \frac{f_n}{f_4} \ . \tag{2.16}$$

The ratios of the six forces on the left hand side of Equation (2.14) to f_4 are found by multiplying both sides of Equation (2.15) by the inverse of the matrix containing the screw information. This can be written as

$$\begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = -\begin{bmatrix} \$_{L1} & \$_{L2} & \$_{L3} & \$_{L5} & \$_{L6} & \$_{Lext} \end{bmatrix}^{-1} \$_{L4} \tag{2.17}$$

The results of Equation (2.17) are combined with the information regarding the potential energy stored in the springs to solve for the forces in the legs and the external force. Knowledge of the forces in the side ties yields the elongation of the springs. The elongation information, combined with the total length of the side ties, gives the amount of cable paid out to each leg. The equation for the total amount of potential energy stored in the three legs (U) is

$$U = \frac{k_4\delta_4^2 + k_5\delta_5^2 + k_6\delta_6^2}{2} \tag{2.18}$$

where δ_i is the deformation of the spring along leg i.

This equation is combined with the formula for the force in a spring,

$$f = k * \delta \tag{2.19}$$

to yield Equation (2.20), which can be written as

$$U = \left(\frac{f_4^2}{k_4} + \frac{f_5^2}{k_5} + \frac{f_6^2}{k_6} \right) \Big/ 2 \ . \tag{2.20}$$

When this is combined with the formula for f_n', then the total potential energy stored in the springs is given in terms of f_4 and the ratios of the other forces as

$$U = \left(\frac{f_4^2}{k_4} + \frac{f_4^2 f_5'^2}{k_5} + \frac{f_4^2 f_6'^2}{k_6} \right) \Big/ 2 \ . \tag{2.21}$$

Solving this equation for f_4 yields

$$f_4 = \sqrt{\frac{2U * k_4 k_5 k_6}{k_5 k_6 + k_4 k_6 f_5'^2 + k_4 k_5 f_6'^2}} \ . \tag{2.22}$$

The values for f_5' and f_6' obtained from Equation (2.17) are used in Equation (2.22) to solve for f_4. This value yields the other six forces that act on the top platform as

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_5 \\ f_6 \\ f_{ext} \end{bmatrix} = f_4 \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} . \tag{2.23}$$

The elongations of the springs are obtained by rearranging Equation (2.19) into the form

$$\delta = \frac{f}{k} \ . \tag{2.24}$$

2.7 Obtain Leg and Cable Lengths

Once the spring deformations are known, the desired information of the reverse analysis can be found using a three-step process. The first step is to obtain the endpoints of each leg in space. This is done by using Equations (2.1), (2.2), and (2.3) in conjunction with Equation (2.5). The second step is to calculate the equations of the lines representing the legs. The last step is to calculate the forces in the side ties, which yields the spring elongations and the cable lengths in those legs.

After the global point vectors are obtained for the joints one equation remains to yield the values of the leg lengths. A leg length is equal to the length of the line segment connecting its two joints. This magnitude is found by using Equation (2.10). This represents the required value for the non-compliant legs, but for the side-ties the length of cable was required. This length is found by subtracting the sum of the spring elongation and the spring free length from the total leg length found in the above equation. The equations for the lengths of the struts and of the cable in each side tie are given below as Equation (2.25) and (2.26).

$$\mathrm{L}_{1,2,3} = \sqrt{\mathrm{l}_{1,2,3}^2 + \mathrm{m}_{1,2,3}^2 + \mathrm{n}_{1,2,3}^2} \tag{2.25}$$

$$lc_{4,5,6} = \sqrt{l_{1,2,3}^2 + m_{1,2,3}^2 + n_{1,2,3}^2} - l_{0_{4,5,6}} - \delta_{4,5,6} \tag{2.26}$$

These equations result in the desired information from the problem statement. The values for the spring free lengths, $l_{0_{4,5,6}}$, are given in the problem statement.

2.8 On the Requirement for an External Wrench

If there was no external wrench applied to the mechanism, the force balance equation could be written as

$$\begin{bmatrix} \mathbf{\$}_{L1} & \mathbf{\$}_{L2} & \mathbf{\$}_{L3} & \mathbf{\$}_{L4} & \mathbf{\$}_{L5} & \mathbf{\$}_{L6} \end{bmatrix} \begin{bmatrix} \mathrm{f}_1 \\ \mathrm{f}_2 \\ \mathrm{f}_3 \\ \mathrm{f}_4 \\ \mathrm{f}_5 \\ \mathrm{f}_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.27}$$

There are two situations in which this equation can hold true. One is the trivial case where all of the forces are zero. The other requires at least one of the lines to be linearly dependant on one or more of the other lines, i.e. the rank of the matrix of line coordinates must be less than six. This can only occur for special configurations of the mechanism's legs and it is said that the mechanism position is singular. That fact limits the equilibrium position and orientation of the top platform to those few states where the line coordinates are linearly dependent. Since the force balance equation is only true at those positions, the device will only be static at those positions. If it is desired for the top platform to attain any desired position and orientation, then the external wrench of the problem statement must be applied.

In space, some linear combination of six independent forces is required to produce a general wrench. This means that as long as the line coordinates of the legs are linearly independent then any external wrench applied to the mechanism can be equilibrated by the summation of forces in the six legs. For clarity, this concept will be illustrated in the plane.

The coordinates of a force in the plane can be described by three variables: l, m, and r. These represent the force's x component, y component, and resulting moment about the z-axis. In the plane, three independent forces are required in some linear combination to produce a general force (the same is true for lines). These three forces, however, cannot create a resultant force of zero magnitude. There are only two cases where the sum of three forces in the plane can equal zero, either the magnitude of each is zero or they intersect at one point. In addition to intersecting at a single point, it is required that the three forces exist in a certain linear combination, but there is no combination of the forces that can yield zero if they do not all intersect at a single point. Three forces, which would result in a zero magnitude force if they intersected at a point, produce a resultant moment about the z-axis when they do not intersect at a single point. That they intersect at a single point indicates that they are linearly dependent.

Linear dependence of lines can be expressed thusly. If a line exists in a set, and can be produced by a linear combination of some other lines in the set, then the set is linearly dependent. It follows that there is not a set of four linearly independent lines in the plane (seven in space). All the linear combinations of any two lines in the plane create a planar pencil of lines through the intersection of the two. That is, every possible line that could pass through the intersection point. Thus if three lines pass through a point they must be linearly dependent. Conversely, a linear combination of two lines cannot result in a general line, only one that passes through their point of intersection. An illustration of a planar pencil is given in Figure 2-2.

If three nonzero forces are to have a resultant of zero, then they must intersect in a point. Thus, they must be linearly dependent. If given the lines of action for two forces in the plane and asked to find a third force which, when summed with the other two, will not have any resultant, then the placement of the third force is limited to cases where it is directed through the point of intersection of the two. This applies to the device under consideration in that it is limited to certain configurations in the absence of an external wrench.

In order for six forces in space to be dependent, though they need not all intersect at a point, there are still restrictions on their arrangement. These restrictions prohibit the tensegrity-based mechanism discussed thus far from having infinite mobility. Infinite mobility is used here in the sense that the mechanism would not be continuously in equilibrium from one arbitrary position and orientation to another. An external wrench is applied so that the forces in the legs can equilibrate with something other than zero. They can equilibrate with a wrench that acts along a general screw when they are in any arrangement *other* than a singularity position.

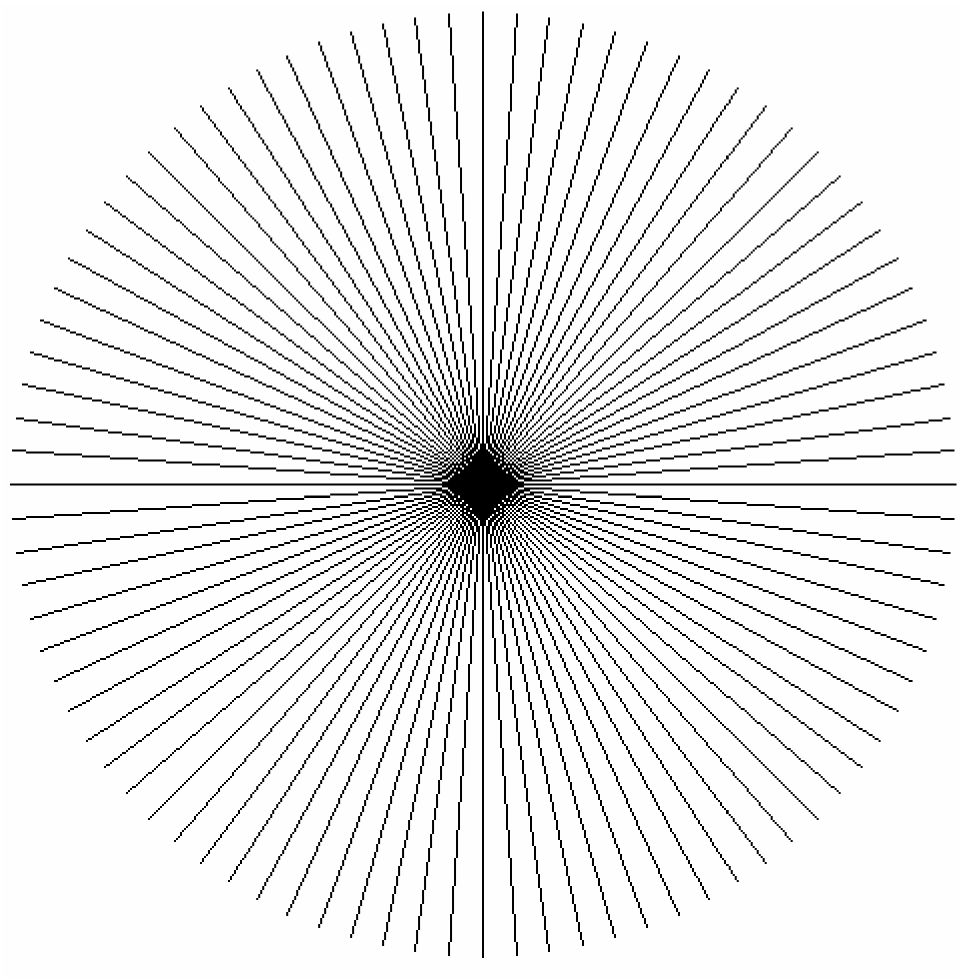

Figure 2-2. Planar Pencil of Lines

2.9 Numerical Examples

2.9.1 Example 1

In example one the top and bottom platforms are of equal size. The joints meet at three points per platform. The three spring constants are equal, as are their free lengths.

There is no pitch to the screw of action of the external wrench. That screw passes through the center of each platform. For a set of values given below, the solution follows.

$l_t = l_b = 20.0$ cm

$\sigma = 0.0$

$k_4 = k_5 = k_6 = 20.0$ N/cm

$l_{0_4} = l_{0_5} = l_{0_6} = 3.0$ cm

$U = 40.0$ N cm

$$ {}^{B}_{T}T = \begin{bmatrix} 0.893 & 0.325 & 0.312 & 8 \\ -0.326 & 0.944 & -0.051 & 5 \\ -0.312 & -0.056 & 0.949 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \$_{ext} = \begin{bmatrix} 0.516 \\ 0.083 \\ 0.853 \\ 4.924 \\ -8.528 \\ -2.147 \end{bmatrix} $$

Incidentally, the given transformation matrix describes translations in the x, y, and z directions of 8, 5, and 18 cm respectively. It also denotes a rotation of 3.1 degrees about its new x-axis, then a rotation of 18.2 degrees about its new y-axis. The final process that the top coordinate system undergoes is a rotation of minus 20 degrees about its z-axis. The external screw has no pitch and lies on a line that passes through the center of each platform.

Using the given values for l_t, l_b, and σ with Equations (2.1), (2.2), and (2.3) yields the point coordinates for the joints of the bottom and top platforms in the bottom and top coordinate systems respectively. Since σ is equal to zero in this example the total joints of each platform lie on only three points. The local coordinates of those points are

$^{B}\mathbf{P}_1 = {}^{T}\mathbf{P}_4 = [0 \quad 0 \quad 0]\text{cm}$,

$^{B}\mathbf{P}_2 = {}^{T}\mathbf{P}_5 = [20 \quad 0 \quad 0]\text{cm}$,

$^{B}\mathbf{P}_3 = {}^{T}\mathbf{P}_6 = [10 \quad 17.32 \quad 0]\text{cm}$.

The preceding coordinates for points four, five, and six are in the top platform coordinate system. To transform these point coordinates into the base coordinate system, they, along with the given transformation matrix, are used in Equations (2.5) and (2.6) to yield the following global coordinates:

$^{B}\mathbf{P}_4 = [8 \quad 5 \quad 18]\text{cm}$,

$^{B}\mathbf{P}_5 = [25.851 \quad -1.52 \quad 11.769]\text{cm}$,

$^{B}\mathbf{P}_6 = [22.559 \quad 18.09 \quad 13.915]\text{cm}$.

The six joint coordinates in the base system can be used in Equation (2.8) to yield the following Plücker coordinates:

$$[\$_{L1} \quad \$_{L2} \quad \$_{L3} \quad \$_{L4} \quad \$_{L5} \quad \$_{L6}] = \begin{bmatrix} 8 & 5.854 & 12.554 & 12 & -15.854 & -22.554 \\ 5 & -1.513 & 0.775 & -5 & 18.833 & -18.096 \\ 18 & 11.769 & 13.917 & 18 & -11.769 & -13.917 \\ 0 & 0 & 241.049 & 0 & -203.84 & 0 \\ 0 & -235.374 & -139.17 & 360 & 117.687 & 0 \\ 0 & -30.259 & -209.696 & -100 & 462.927 & 0 \end{bmatrix}$$

Taking the magnitudes of the lines given above gives the total leg lengths. This is accomplished using equation (2.10). Those leg lengths are

$L_1 = 20.322$ cm ,

$L_2 = 13.231$ cm ,

$L_3 = 18.759$ cm ,

L_4 = 22.204 cm ,

L_5 = 27.286 cm ,

L_6 = 32.091 cm.

The line coordinates in this matrix are unitized using Equations (2.9) and (2.10). The unitized coordinates of these lines and the external screw are then used in the force balance equation, Equation (2.12) as

$$\begin{bmatrix} 0.394 & 0.442 & 0.669 & 0.54 & -0.581 & -0.703 & 0.516 \\ 0.246 & -0.114 & 0.041 & -0.225 & 0.69 & -0.564 & 0.083 \\ 0.886 & 0.889 & 0.742 & -0.811 & -0.431 & -0.434 & 0.853 \\ 0 & 0 & 12.85 & 0 & -7.47 & 0 & 4.924 \\ 0 & -17.790 & -7.419 & 16.214 & 4.313 & 0 & -8.528 \\ 0 & -2.287 & -11.179 & -4.504 & 16.966 & 0 & -2.144 \end{bmatrix} \begin{bmatrix} \mathrm{f}_1 \\ \mathrm{f}_2 \\ \mathrm{f}_3 \\ \mathrm{f}_4 \\ \mathrm{f}_5 \\ \mathrm{f}_6 \\ \mathrm{f}_{\mathrm{ext}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

These values are rearranged into the form of Equation (2.14) to yield

$$\begin{bmatrix} 0.394 & 0.442 & 0.669 & -0.581 & -0.703 & 0.516 \\ 0.246 & -0.114 & 0.041 & 0.69 & -0.564 & 0.083 \\ 0.886 & 0.889 & 0.742 & -0.431 & -0.434 & 0.853 \\ 0 & 0 & 12.85 & -7.47 & 0 & 4.924 \\ 0 & -17.790 & -7.419 & 4.313 & 0 & -8.528 \\ 0 & -2.287 & -11.179 & 16.966 & 0 & -2.144 \end{bmatrix} \begin{bmatrix} \mathrm{f}_1' \\ \mathrm{f}_2' \\ \mathrm{f}_3' \\ \mathrm{f}_5' \\ \mathrm{f}_6' \\ \mathrm{f}_{ext}' \end{bmatrix} = \begin{bmatrix} -0.54 \\ 0.225 \\ 0.811 \\ 0 \\ -16.214 \\ 4.504 \end{bmatrix}.$$

Both sides of this equation are multiplied by the inverse of the 6×6 matrix to yield the following values:

$$\begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} 1.583 \\ 1.821 \\ 1.91 \\ 1.41 \\ 1.386 \\ -2.846 \end{bmatrix}.$$

The value of f_4 was found by placing values above and the given spring data in Equation (2.22). The resulting value is

$f_4 = 18.147$ N

This value is used along with Equation (2.15) to yield values for the remaining forces acting on the top platform.

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_5 \\ f_6 \\ f_{ext} \end{bmatrix} = \begin{bmatrix} 28.73 \\ 33.046 \\ 34.664 \\ 25.584 \\ 24.822 \\ -51.649 \end{bmatrix} \text{N.}$$

Equation (2.26) can be used to find the amount of cable paid out to each side tie by using the forces in the side ties (f_4, f_5, and f_6), the given spring data, and the leg lengths found above. The results of that process are

$lc_4 = 18.297$ cm ,

$lc_5 = 23.007$ cm ,

$lc_6 = 27.850$ cm .

These values complete the reverse analysis solution for a tensegrity-based parallel-platform device.

2.9.2 Example 2

The given data for this example are identical to those of Example 1 except for the value of the potential energy.

$U = 87.20$ N cm

The solution in this example proceeds in the same manner as that of the previous one. The effect of the larger potential energy is not noticeable in the solution until the value for f_4 is calculated as

$f_4 = 26.794$ N

The ratios of the remaining forces that act on the top platform to f_4 are equal to those found in Example 1. The magnitudes of the remaining forces are

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_5 \\ f_6 \\ f_{ext} \end{bmatrix} = \begin{bmatrix} 42.42 \\ 48.792 \\ 51.181 \\ 37.774 \\ 36.649 \\ -76.259 \end{bmatrix}$$

The length of cable paid out to each side tie is found in an identical manner to that found in Example 1. Those lengths are given below.

$lc_4 = 17.864$ cm

$lc_5 = 22.397$ cm

$lc_6 = 27.259$ cm

2.9.3 Example 3

The supplied information in Example 3 is identical to that in Example 1 except for the screw of action of the external wrench. This unitized screw is given below.

$$\$_{ext} = \begin{bmatrix} 0.032 \\ -0.599 \\ 0.8 \\ 10.209 \\ -10.474 \\ -8.032 \end{bmatrix}$$

This screw has the value 0.17 cm for its pitch.

The data for this screw are combined below with the unitized line coordinates found in Example 1 to make an equation of the form of Equation (2.14) as

$$\begin{bmatrix} 0.394 & 0.442 & 0.669 & -0.581 & -0.703 & 0.032 \\ 0.246 & -0.114 & 0.041 & 0.69 & -0.564 & -0.599 \\ 0.886 & 0.889 & 0.742 & -0.431 & -0.434 & 0.8 \\ 0 & 0 & 12.85 & -7.47 & 0 & 10.209 \\ 0 & -17.790 & -7.419 & 4.313 & 0 & -10.474 \\ 0 & -2.287 & -11.179 & 16.966 & 0 & -8.032 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} -0.54 \\ 0.225 \\ 0.811 \\ 0 \\ -16.214 \\ 4.504 \end{bmatrix}.$$

This equation is solved to yield

$$\begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} 18.371 \\ 12.816 \\ 23.074 \\ 7.91 \\ 38.488 \\ -23.256 \end{bmatrix}$$

These yield the value

$$f_4 = 7.491\ \text{N} \ .$$

This force magnitude allows for the solution to complete through the application of first Equation (2.15) then Equation (2.26). The amount of cable paid out to each compliant leg is listed below.

lc_4 = 18.829 cm

lc_5 = 23.890 cm

lc_6 = 27.167 cm

2.9.4 Example 4

In example four the top and bottom platforms are of unequal size. The end joints are fixed at six points per platform. The three spring constants are also unequal. Their free lengths are the same. The screw of action of the external wrench has a pitch of 0.3. For a set of values given below, the solution follows.

l_t = 18.0 cm

l_b = 22.0 cm

σ_t = 0.07

σ_b = 0.16

k_4 = 18.0 N/cm

k_5 = 23.0 N/cm

k_6 = 30.0 N/cm

$$l_{0_4} = l_{0_5} = l_{0_6} = 3.0 \text{ cm}$$

U = 150.0 N cm

$$ {}^{B}_{T}T = \begin{bmatrix} 0.669 & -0.743 & 0 & 5 \\ 0.732 & 0.659 & -0.174 & -6 \\ 0.129 & 0.116 & 0.985 & 16 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$\$_{ext} = \begin{bmatrix} -0.389 \\ -0.167 \\ 0.906 \\ 8.203 \\ -11.309 \\ 1.773 \end{bmatrix}$$

Using the given values for l_t, l_b, σ_t, and σ_b with Equations (2.1), (2.2), and (2.3) yields the point coordinates for the joints of the bottom and top platforms in the bottom and top coordinate systems respectively as

${}^{B}\mathbf{P}_1 = {}^{T}\mathbf{P}_4 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \text{cm}$,

${}^{B}\mathbf{P}_{42} = \begin{bmatrix} 18.48 & 0 & 0 \end{bmatrix} \text{cm}$,

${}^{B}\mathbf{P}_2 = \begin{bmatrix} 22 & 0 & 0 \end{bmatrix} \text{cm}$,

${}^{B}\mathbf{P}_{53} = \begin{bmatrix} 12.76 & 16.004 & 0 \end{bmatrix} \text{cm}$,

${}^{B}\mathbf{P}_3 = \begin{bmatrix} 11 & 19.053 & 0 \end{bmatrix} \text{cm}$,

${}^{B}\mathbf{P}_{61} = \begin{bmatrix} 1.76 & 3.048 & 0 \end{bmatrix} \text{cm}$,

${}^{T}\mathbf{P}_{25} = \begin{bmatrix} 16.74 & 0 & 0 \end{bmatrix} \text{cm}$,

${}^{T}\mathbf{P}_5 = \begin{bmatrix} 18 & 0 & 0 \end{bmatrix} \text{cm}$,

${}^{T}\mathbf{P}_{36} = \begin{bmatrix} 9.63 & 14.497 & 0 \end{bmatrix} \text{cm}$,

${}^{T}\mathbf{P}_3 = \begin{bmatrix} 9 & 15.588 & 0 \end{bmatrix} \text{cm}$,

${}^{T}\mathbf{P}_{14} = \begin{bmatrix} 0.63 & 1.091 & 0 \end{bmatrix} \text{cm}$,

To transform point coordinates from the system that is relative to the top platform into the base coordinate system, they, along with the given transformation matrix, are used in Equations (2.5) and (2.6) to yield the following global coordinates:

$$ {}^{B}\mathbf{P}_4 = [5 \quad -6 \quad 16]\,\text{cm}, $$

$$ {}^{B}\mathbf{P}_{25} = [16.201 \quad 6.251 \quad 18.16]\,\text{cm}, $$

$$ {}^{B}\mathbf{P}_5 = [17.044 \quad 7.173 \quad 18.323]\,\text{cm}, $$

$$ {}^{B}\mathbf{P}_{36} = [0.670 \quad 10.601 \quad 18.927]\,\text{cm}, $$

$$ {}^{B}\mathbf{P}_6 = [-0.562 \quad 10.859 \quad 18.973]\,\text{cm}. $$

$$ {}^{B}\mathbf{P}_{14} = [4.611 \quad -4.82 \quad 16.208]\,\text{cm}. $$

The six joint coordinates in the base system can be used in Equation (2.8) to yield the following Plücker coordinates:

$$ [\$_{L1} \quad \$_{L2} \quad \$_{L3} \quad \$_{L4} \quad \$_{L5} \quad \$_{L6}] = \begin{bmatrix} 4.611 & -5.799 & -10.33 & 13.48 & -4.284 & 2.322 \\ -4.82 & 6.251 & -8.452 & 6 & 8.831 & -7.811 \\ 16.208 & 18.16 & 18.927 & -16 & -18.323 & -18.973 \\ 0 & 0 & 360.611 & 0 & -293.241 & -57.836 \\ 0 & -399.525 & -208.199 & 295.68 & 233.799 & 33.392 \\ 0 & 137.527 & 103.842 & 110.88 & 181.248 & -20.826 \end{bmatrix} $$

Taking the magnitudes of the lines given above gives the total leg lengths. This is accomplished using equation (2.10). Those leg lengths are

L_1 = 17.527 cm ,

L_2 = 20.062 cm ,

L_3 = 23.16 cm ,

L_4 = 21.765 cm ,

L_5 = 20.786 cm ,

L_6 = 20.648 cm.

The line coordinates in this matrix are unitized using Equations (2.9) and (2.10). The unitized coordinates of these lines and the external screw are then used in the force balance equation, Equation (2.12) as

$$\begin{bmatrix} 0.263 & -0.289 & -0.446 & 0.619 & -0.206 & 0.112 & -0.389 \\ -0.275 & 0.312 & -0.365 & 0.276 & 0.425 & -0.378 & -0.167 \\ 0.925 & 0.905 & 0.817 & -0.735 & -0.881 & -0.919 & 0.906 \\ 0 & 0 & 15.571 & 0 & -14.108 & -2.801 & 8.203 \\ 0 & -19.914 & -8.99 & 13.585 & 11.248 & 1.617 & -11.309 \\ 0 & 6.855 & 4.484 & 5.094 & 8.72 & -1.009 & 1.773 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_{ext} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

This data is rearranged into the form of Equation (2.14) to yield

$$\begin{bmatrix} 0.394 & 0.442 & 0.669 & -0.581 & -0.703 & 0.516 \\ 0.246 & -0.114 & 0.041 & 0.69 & -0.564 & 0.083 \\ 0.886 & 0.889 & 0.742 & -0.431 & -0.434 & 0.853 \\ 0 & 0 & 12.85 & -7.47 & 0 & 4.924 \\ 0 & -17.790 & -7.419 & 4.313 & 0 & -8.528 \\ 0 & -2.287 & -11.179 & 16.966 & 0 & -2.144 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} -0.54 \\ 0.225 \\ 0.811 \\ 0 \\ -16.214 \\ 4.504 \end{bmatrix}.$$

Both sides of this equation are multiplied by the inverse of the 6×6 matrix to yield the following values:

$$\begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} -0.234 \\ -0.711 \\ -1.998 \\ 0.19 \\ 0.553 \\ 4.309 \end{bmatrix}.$$

The value of f_4 was found by placing values above and the given spring data in Equation (2.22). The resulting value is

$f_4 = 66.757$ N

This value is used along with Equation (2.15) to yield values for the remaining forces acting on the top platform.

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_5 \\ f_6 \\ f_{ext} \end{bmatrix} = \begin{bmatrix} -15.596 \\ -47.435 \\ -133.406 \\ 12.673 \\ 36.92 \\ 287.642 \end{bmatrix} \text{N.}$$

Equation (2.26) can be used to find the amount of cable paid out to each side tie by using the forces in the side ties (f_4, f_5, and f_6), the given spring data, and the leg lengths found above. The results of that process are

$lc_4 = 15.056$ cm ,

$lc_5 = 17.235$ cm ,

$lc_6 = 16.417$ cm .

These values complete the reverse analysis solution for a tensegrity-based parallel-platform device.

CHAPTER 3
TENSEGRITY-BASED PARALLEL PLATFORM WITH SEVENTH LEG

3.1 Application of a Seventh Leg

This chapter introduces the concept of adding a seventh leg to the mechanism. The seventh leg consists of a prismatic connector attached to the top and bottom platforms by a Hooke and ball-and-socket joint respectively. The purpose of this leg is to apply an “external force” to the platform of the tensegrity-based 3-3 in-parallel platform device. The six other legs are to equilibrate with this force. As shown in Chapter two, section nine, a pure applied force is sufficient for the device to obtain a general configuration.

For the external wrench to model a pure force, its pitch must equal zero. This means that a force applied by the seventh leg as described above is sufficient for the obtaining of a solution. Though the magnitude of this force is not controllable with a prismatic joint, the length of the seventh leg need only be set and the proper resultant force will arise naturally. The leg lengths for this type of mechanism are found using the method of the previous chapter, substituting the unitized Plücker Coordinates of the seventh leg for the external screw. Placement of the end-joints of the seventh leg is important

Chapter two, section nine, demonstrates that solutions for identical configurations (potential energy, spring and platform characteristics, and position and orientation) existed for different screws along which the external wrench acted. This quality of having multiple possible external screws of action for a given configuration implies that the distinction of the external screw of action is not paramount to the solution. Section eight of the same chapter however describes to what extent this distinction is important.

The external screw of action cannot be placed in such a way that it is dependent on any five of the legs. This applies to the placement of the seventh leg of the device. If the seventh leg is a slider joint, then in a given position and orientation of the device, the seventh leg must lie along a line such that the determinant of the six by six matrix in Equation (2.13) is not zero. If the determinant is zero, then the analysis cannot be solved in the manner previously discussed, because the matrix cannot be inverted. It is shown in the next section of this chapter that if the determinant equals zero, then either the characteristics of the seventh leg can be changed or the value of σ can be changed, and a solution will exist. A computer program was written that would both create depictions of, and provide solutions for, seven-legged tensegrity-based parallel platform devices, provided that they are in non-singular configurations. If the matrix discussed above is singular, then the device will be rendered, but no solution will be produced.

3.2 Numerical Examples

Three renderings of a seven-legged tensegrity-based parallel-platform device, Figure 3-1, 3-2, and 3-3, are shown in **3.2.1**, **3.2.2**, and **3.2.3**. The blue-and-grey legs in the figures represent the compliant ties, the blue portions depicting the springs. The green-and-yellow legs are thus the struts. The device in these drawings is in the same configuration except that in Figure 3-2 the seventh leg acts along a screw of non-zero pitch, and in Figure 3-3 the joints are offset 20% of the platform length.

The seventh leg is means of applying the required external wrench to the device. For this reason its properties are represented in the following examples in the same way those of the external force were denoted in the examples of the previous chapter. For example the magnitude of the force acting along the seventh leg is given by the variable f_{ext}.

3.2.1 Example 1

In this example the device is translated in the three principal directions and not rotated about any axes. The transformation matrix for this process is

$$ {}^{B}_{T}T = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 16 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

The platform lengths, the joint offset, and the spring data are given here:

$l_t = l_b = 20.0$ cm

$\sigma = 0.0$

$k_4 = k_5 = k_6 = 20.0$ N/cm

$l_{0_4} = l_{0_5} = l_{0_6} = 3.0$ cm

$U = 40.0$ N cm

The end-joints for the seventh leg of this device are placed in the center of each platform. The point coordinates of these joints, $\mathbf{P}_{ext_B}$ and $\mathbf{P}_{ext_T}$, in a system local to the platform of each joint, are

$$ {}^{B}\mathbf{P}_{ext_B} = {}^{T}\mathbf{P}_{ext_T} = \begin{bmatrix} 10 & 5.77 & 0 \end{bmatrix} \text{ cm.} $$

The point vector of the top joint is then transformed into global coordinates by using Equation (2.5). The resulting point coordinates are

$$ {}^{B}\mathbf{P}_{ext_T} = \begin{bmatrix} 18 & 13.77 & 16 \end{bmatrix} \text{ cm.} $$

In this example the force in the seventh leg acts on the top platform along a screw of zero pitch. Thus the equation of this screw, when unitized, is equal to the unitized Plücker coordinates of a line along which the leg lies. Those coordinates are obtained from Equation (2.8) and ca be written as

$$\$_{ext} = \begin{bmatrix} 0.408 \\ 0.408 \\ 0.816 \\ 4.711 \\ -8.165 \\ 1.727 \end{bmatrix}.$$

Using the given values for l_t, l_b, and σ with Equations (2.1), (2.2), and (2.3) yields the point coordinates for the joints of the bottom and top platforms in the bottom and top coordinate systems respectively. Since σ is equal to zero in this example the total joints of each platform lie on only three points. The local coordinates of those points are

$${}^{B}\mathbf{P}_1 = {}^{T}\mathbf{P}_4 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \text{cm},$$

$${}^{B}\mathbf{P}_2 = {}^{T}\mathbf{P}_5 = \begin{bmatrix} 20 & 0 & 0 \end{bmatrix} \text{cm},$$

$${}^{B}\mathbf{P}_3 = {}^{T}\mathbf{P}_6 = \begin{bmatrix} 10 & 17.32 & 0 \end{bmatrix} \text{cm}.$$

The preceding coordinates for points four, five, and six are in the top platform coordinate system. To transform these point coordinates into the base coordinate system they, along with the given transformation matrix, are used in Equations (2.5) and (2.6) to yield

$${}^{B}\mathbf{P}_4 = \begin{bmatrix} 8 & 8 & 16 \end{bmatrix} \text{cm},$$

$${}^{B}\mathbf{P}_5 = \begin{bmatrix} 28 & 8 & 16 \end{bmatrix} \text{cm},$$

$${}^{B}\mathbf{P}_6 = \begin{bmatrix} 18 & 25.32 & 16 \end{bmatrix} \text{cm}.$$

The joint coordinates of legs one through six can be used in Equation (2.8) to yield their Plücker coordinates, which are expressed in matrix form as

$$[\$_{L1} \quad \$_{L2} \quad \$_{L3} \quad \$_{L4} \quad \$_{L5} \quad \$_{L6}] = \begin{bmatrix} 8 & 8 & 8 & 12 & -18 & -18 \\ 8 & 8 & 8 & -8 & 9.32 & -25.32 \\ 16 & 16 & 16 & -16 & -16 & -16 \\ 0 & 0 & 277.128 & 0 & -277.128 & 0 \\ 0 & -320 & -160 & 320 & 160 & 0 \\ 0 & 160 & -58.564 & -160 & 404.974 & 0 \end{bmatrix}.$$

The line coordinates in this matrix are unitized using Equations (2.9) and (2.10). The unitized coordinates are then used in the force balance equation, Equation (2.12) as

$$\begin{bmatrix} 0.408 & 0.408 & 0.408 & 0.557 & -0.697 & -0.515 & 0.408 \\ 0.408 & 0.408 & 0.408 & -0.371 & 0.361 & -0.725 & 0.408 \\ 0.816 & 0.816 & 0.816 & -0.743 & -0.62 & -0.458 & 0.816 \\ 0 & 0 & 14.142 & 0 & -10.731 & 0 & 4.711 \\ 0 & -16.33 & -8.165 & 14.856 & 6.196 & 0 & -8.165 \\ 0 & 8.165 & -2.989 & -7.428 & 15.682 & 0 & 1.727 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_{ext} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

This data is rearranged into the form of Equation (2.14) to yield

$$\begin{bmatrix} 0.408 & 0.408 & 0.408 & -0.697 & -0.515 & 0.408 \\ 0.408 & 0.408 & 0.408 & 0.361 & -0.725 & 0.408 \\ 0.816 & 0.816 & 0.816 & -0.620 & -0.458 & 0.816 \\ 0 & 0 & 14.142 & -10.731 & 0 & 4.711 \\ 0 & -16.33 & -8.165 & 6.196 & 0 & -8.165 \\ 0 & 8.165 & -2.989 & 15.682 & 0 & 1.727 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} -0.557 \\ 0.371 \\ 0.743 \\ 0 \\ -14.856 \\ 7.428 \end{bmatrix}.$$

The square matrix above cannot be inverted, thus there is no solution to this equation.

The results of inputting this example into the program described earlier are shown below in Figure 3-1.

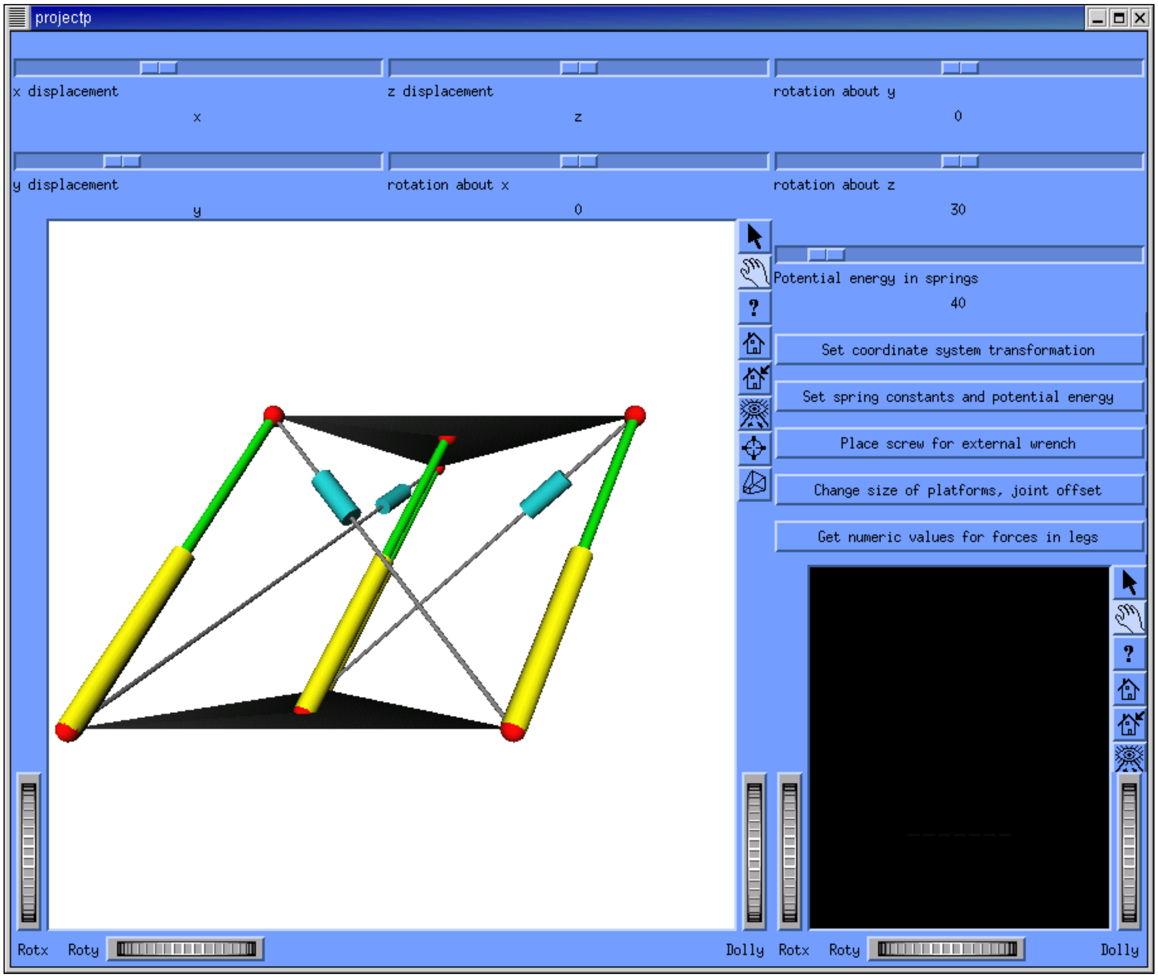

Figure 3-1. Seven-Legged Tensegrity-Based Device in Singularity Position

If a solution for this setup existed then the black window on the right side of the above figure would show a pictorial representation of the relative sizes of the force magnitudes in the legs. The next figure shows the device in the same position and orientation, but that a solution exists for the input of the next example.

3.2.2 Example 2

The relative magnitudes of the forces in the legs appear in the small window. From left to right they represent the force magnitudes $f_1, f_2, f_3, f_4, f_5, f_6$, and f_{ext}. A solution exists in this example because the screw of action for the seventh leg is different from that in 3.2.1 Example 1. The information given in the problem description is the same for this example as it is for Example 1, except for the pitch of the external screw, which is equal to 0.67.

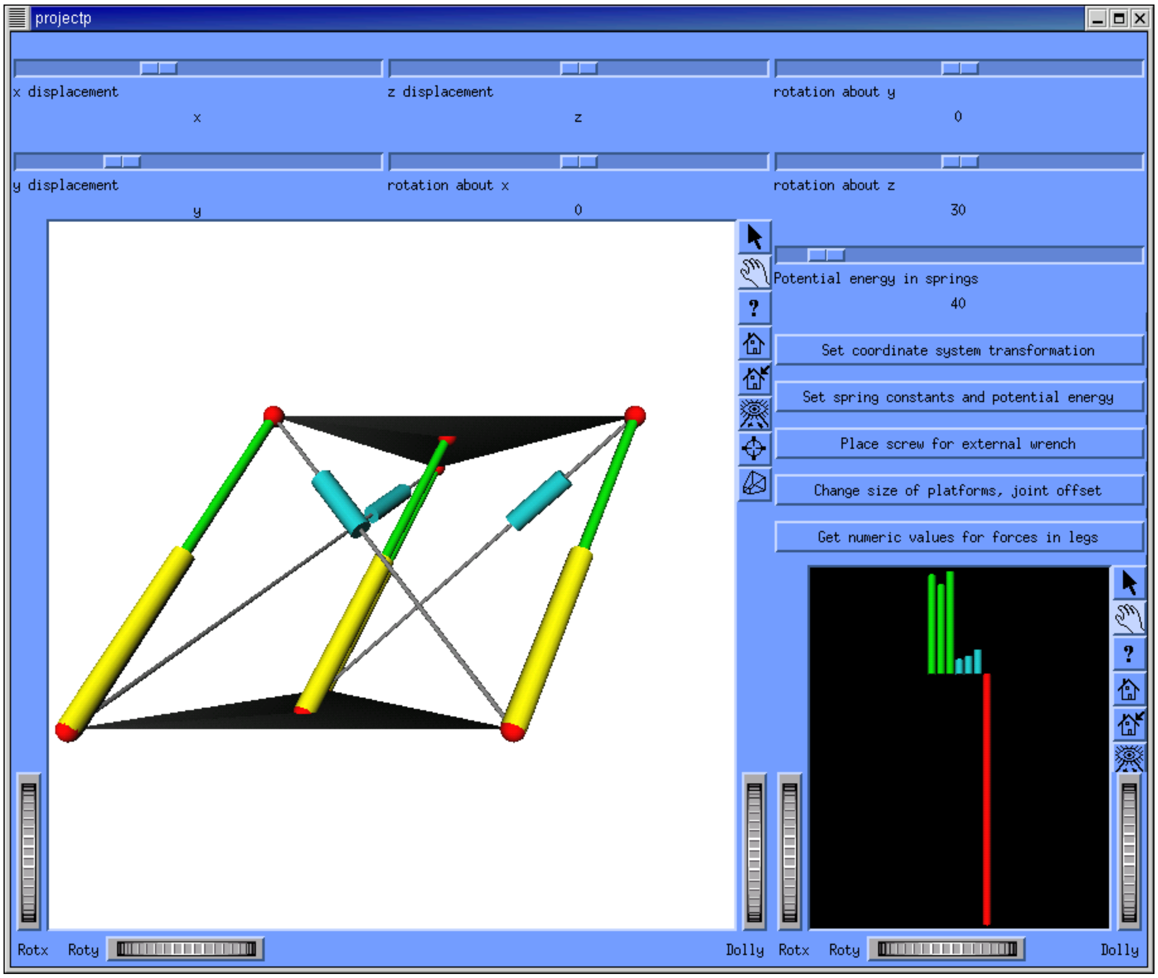

Figure 3-2. Seven-Legged Device with Non-Zero Pitch on the Seventh Screw

Here the Plücker coordinates for the line along which the leg lies are the same, but they are not equal to the equation of the external screw of action as shown in the previous example. In this example the unitized screw is found by using Equation (2.12)

$$\$_{ext} = \begin{bmatrix} 0.408 \\ 0.408 \\ 0.816 \\ 4.958 \\ -7.891 \\ 2.274 \end{bmatrix}.$$

This value is used with the unitized line coordinates of the other six legs from Example 1 to yield

$$\begin{bmatrix} 0.408 & 0.408 & 0.408 & -0.697 & -0.515 & 0.408 \\ 0.408 & 0.408 & 0.408 & 0.361 & -0.725 & 0.408 \\ 0.816 & 0.816 & 0.816 & -0.620 & -0.458 & 0.816 \\ 0 & 0 & 14.142 & -10.731 & 0 & 4.985 \\ 0 & -16.33 & -8.165 & 6.196 & 0 & -7.891 \\ 0 & 8.165 & -2.989 & 15.682 & 0 & 2.274 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} -0.557 \\ 0.371 \\ 0.743 \\ 0 \\ -14.856 \\ 7.428 \end{bmatrix}.$$

which is in the form of Equation (2.14).

This square matrix can be inverted, and both sides of the equation are multiplied by that inverse to yield

$$\begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_5' \\ f_6' \\ f_{ext}' \end{bmatrix} = \begin{bmatrix} 7.583 \\ 6.927 \\ 7.818 \\ 1.199 \\ 1.622 \\ -19.598 \end{bmatrix}.$$

This information is used with the spring data in Equation (2.22) to find

$f_4 = 17.766$ N .

The spring elongations and leg lengths can then be found as described in chapter two, section seven and illustrated in chapter two, section nine.

3.2.3 Example 3

The following example is identical to the first example of this chapter except that the joint offset is not zero. Instead

$\sigma = 0.2$

The results of inputting the data of this case into the previously mentioned program are illustrated below in Figure 3-3.

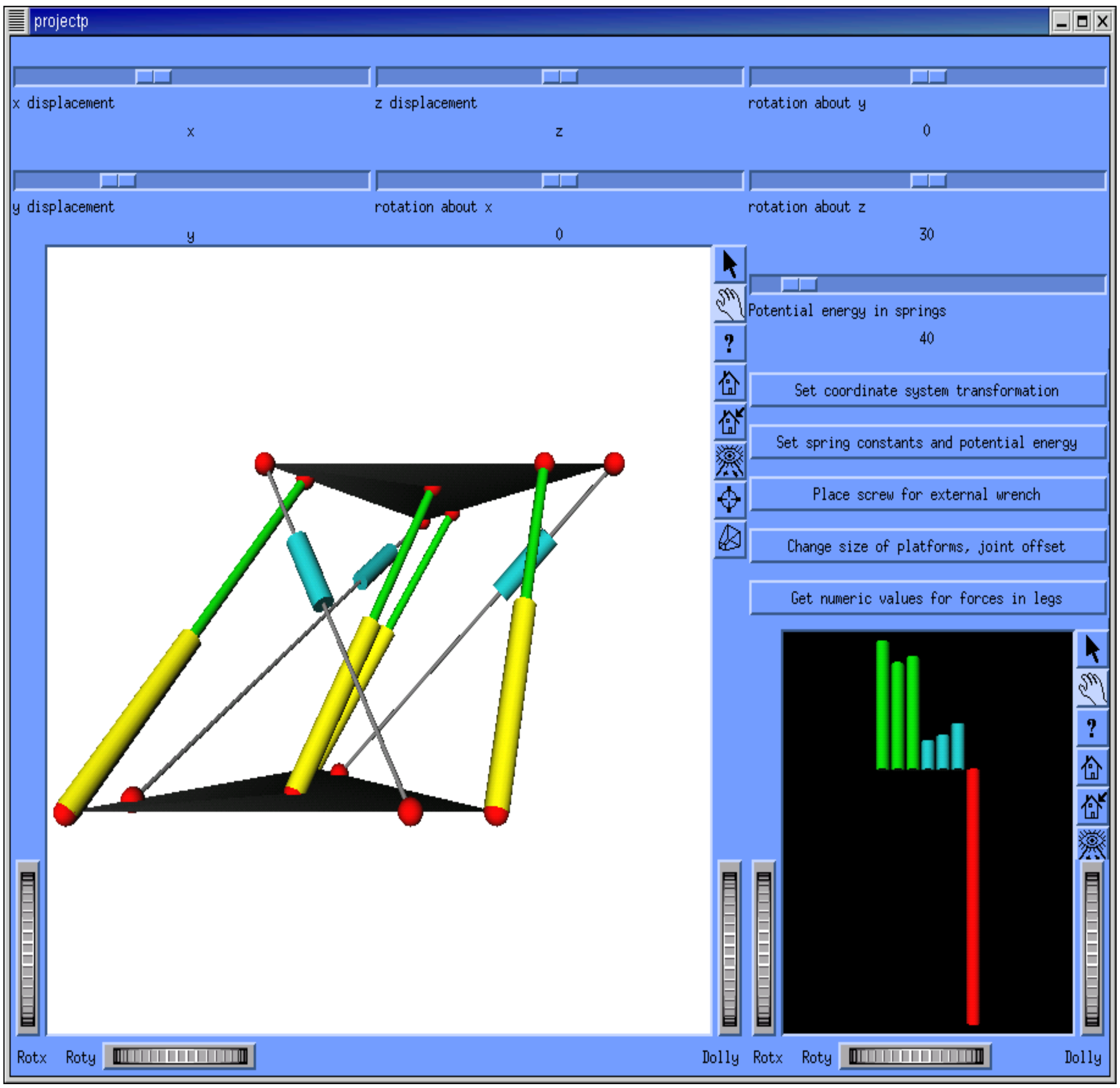

Figure 3-3. Seven-Legged Device with 20% Joint Offset

The relative magnitudes of the forces in the seven legs can again be seen in the black window on the right side of the figure above. Their values are

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_{ext} \end{bmatrix} = \begin{bmatrix} 80.616 \\ 66.933 \\ 70.859 \\ 17.889 \\ 21.337 \\ 28.718 \\ -160.997 \end{bmatrix} \mathrm{N}$$

CHAPTER 4
CONCLUSIONS

A reverse-analysis solution method for a device that is similar to a 3-3 in-parallel platform is explored. This device differed from a 3-3 in-parallel platform in that it is based on the principle of tensegrity. This characteristic allows the device to be varied in position and orientation as well with regards to its compliance characteristics. This device also differs in that it requires the application of an external wrench to achieve a general position and orientation. This paper discusses the need for that wrench

It is put forward in this paper that, for a given position and orientation and with a given internal potential energy in the springs, there are an infinite number of external wrenches with which the six legs of the mechanism can equilibrate. It is shown that the potential energy can vary while the device remains in a constant position and orientation. The solution method of this paper is used on several tensegrity-based devices of varying platform and spring characteristics. This shows that the solution method is applicable for a general tensegrity-based 3-3 parallel platform device.

A different parallel-platform device is proposed. It is based on tensegrity and has four non-compliant struts and three compliant ties. The placement of the seventh leg's joints on the platforms is shown there to be of importance to the feasibility of the solution. It is shown that the singularity problem arising from the seventh leg's location can be overcome either by changing the joint offset, or by altering the pitch of the screw of action of the seventh leg.

There are facets of this device yet unstudied. Its compliance characteristics pose an interesting puzzle. A beneficial solution would be that of the ideal arrangement of leg end joints. Also, the forward analysis of this device must be considered. Here the mechanism dimensions, spring constants, and spring free lengths would be known together with the length of the non-compliant legs and the length of the non-compliant strings that are in series with the springs. The objective would be to determine all the possible positions and orientation of the top platform along with the associated potential energy at this equilibrium position.

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BIOGRAPHICAL SKETCH

Mr. Matthew Quincy Marshall was born in 1978 in DeLand, Florida. In 2001 he received a Bachelor of Science degree in mechanical engineering at the University of Florida. He worked for Exponent Failure Analysis Corp., as an intern. He returned to the University of Florida in August 2001 to garner his Master of Science degree in mechanical engineering.