A Review of a Family of Self-Deploying Tensegrity Structures with Elastic Ties

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ABSTRACT—A review of a family of tensegrity structures that self-deploy from a stowed or packed configuration is presented. In the packed configuration, the mechanism is of a cylindrical form with the struts lying side by side. Such structures may be applied in the deployment of antennas in space and in the rapid deployment of shelters or tents. This family of structures evolved from a study of in-parallel platforms with compliant legs or connectors. A number of relevant references are cited.

1. Introduction

This paper is an account of the evolution of a family of tensegrity structures with elastic ties. The introduction of elastic ties into what are defined as antiprisms by Tobie (1967) and tensegrity prisms by Gabriel (1997) enables them to self-deploy from a stowed or packed configuration, which is of a cylindrical form with the struts lying side by side. When such a structure is released from its stowed position, it self-deploys and reaches a position of minimum potential energy.

One major application of these novel self-deployable structures is in the deployment of antennas in space. An example is illustrated in Figure 1. Incorporated in the base and top are a pair of octagonal-shaped antennas. The self-deployable structure illustrated in Figure 2 is suited to house a single pentagonal-shaped antenna in its base.

Another major application is in self-deploying shelters or tents, which could house personnel and store equipment. Figure 2 illustrates a self-deployable teepee with a pentagonal base. It is important to recognize that these types of structures could be used as self-deployable cells or units, which could be joined together by telescoping struts. In this way, much larger and different-shaped shelters could be assembled relatively easily in a short amount of time.

The term tensegrity was coined by R. Buckminster Fuller (1960). (See Sadao [1996] for an invention by one of his students, K. Snelson, in 1948; see also Pugh [1976a, 1976b], Fuller [1975], Pearce [1978], and Roth and Whiteley [1981].) It stems from a combination of tension and integrity and is used to describe structures that consist of an assemblage of ties and struts, which are in tension and compression. No pair of struts is connected or touches. In the plane, each strut is connected to a pair of ties, whereas in three dimensions, each strut is connected to three noncoplanar ties as illustrated in Figure 3.

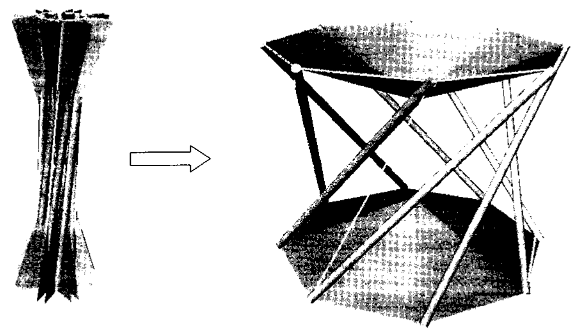

Figure 1. Self-Deployable Tensegrity Structure

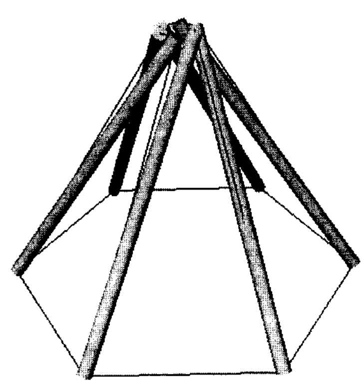

Figure 2. Self-Deployable Tensegrity Structure with a Pentagonal Base

2. The Evolution of a Family of Self-Deploying Tensegrity Structures

This section is a brief account of the evolution of a family of tensegrity structures with compliant ties, which en-

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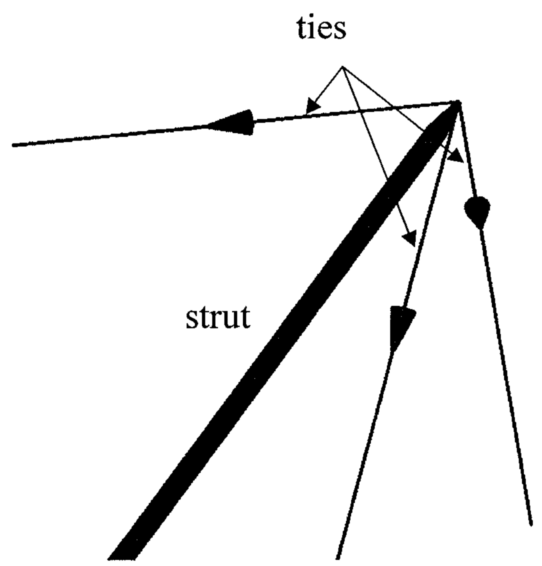


Figure 3. Struts and Ties

able them to self-deploy from a stowed or packed configuration. The evolution is summarized in Figures 4a-4d. It essentially begins with the analysis of in-parallel manipulators (see Figure 4a). Compliance within the legs of these structures leads to the replacement of alternating legs with ties (Figures 4b and 4c). This concept is then expanded to higher order structures (3-3 through 4-4 and 6-6), with the platform upper and lower "surfaces" replaced by ties and deformable materials. It is these deformable surfaces that can act as antenna or tent surfaces.

2.1. In-Parallel Manipulators

An in-parallel manipulator consists of a top (moving) platform connected to a base (fixed) platform by six legs (or connectors), each of which is an S-P-S kinematic chain as illustrated in Figure 4a. The letters S and P denote ball-and-socket and prismatic joints, respectively. Each leg can rotate about a line connecting corresponding points in the base and top platforms. Such motions cannot of course affect the gross motion of the top platform. The term in-parallel was coined by Hunt (1983) to denote that all the connectors consist of the same sequence of kinematic pairs. The top platform has in general six linearly independent instantaneous freedoms measured relative to the base and is said to have 6 degrees of freedom. Typically, the six prismatic joints (one in each leg) are actuated to position and orient the top platform as desired. When all the prismatic pairs are locked, the platform is a structure.

There are multitudes of in-parallel mechanisms with S-P-S connectors, and it is convenient to label them by counting the number of connecting points in the base and the top. Figure 4a illustrates a 6-6 platform, whereas Figure 5 illustrates a 3-3 platform (an octahedron with three connecting points in the base and top platforms), a 6-3 platform, and a regular and a special 6-6 device. The 6-3 device is a kinematic model of the Stewart (1965) platform. It was Stewart who suggested the potential of the 6-3 device as a flight simulator, and this technology has evolved to a highly sophisticated level since the mid-1960s.

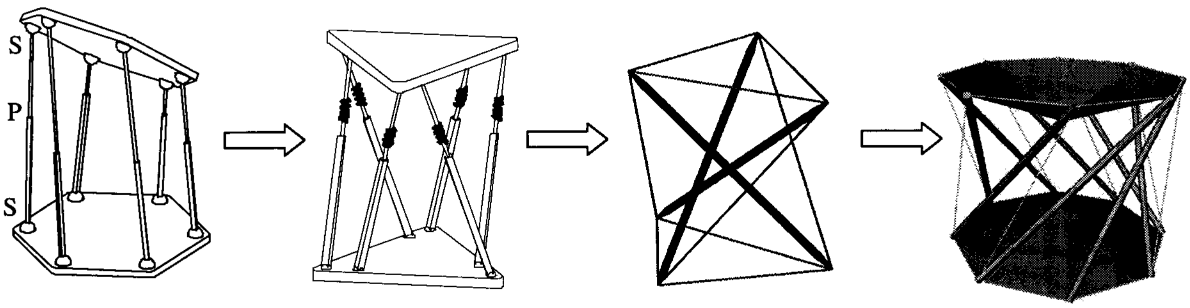
The study of in-parallel platforms largely lay dormant in academia until Fichter (1986) published a paper on the theory and construction of the Stewart-Gough platform. Whereas the so-called forward and reverse finite displacement analyses of serial manipulators had been heavily researched over a period of some 20 years, nothing was known about the difficult forward analysis of in-parallel manipulators: "It is required to compute all possible assembly configurations of the manipulator given the dimensions of the top and base platforms together with the six leg lengths." Kinematicians prefer closed-form solutions as opposed to iterative searches that may fail to find all the real solutions. Briefly, a closed-form solution is expressed by a polynomial in a single variable, which is usually obtained by algebraic elimination from a set of equations in several unknowns. Care must be taken to perform a single elimination (see Salmon, 1876) to avoid obtaining a high-degree polynomial that contains unwanted extraneous roots.

Griffis and Duffy (1989) were the first to obtain an 8th-degree polynomial for the forward solution of the octahedral 3-3 platform and the Stewart-Gough 6-3 platform (see Figure 5). There are a maximum of eight real assembly configurations above the base platform and eight reflections through the base platform. These results were later independently confirmed by Nanua et al. (1990), who also obtained an 8th-degree polynomial using a different problem formulation.

It was becoming clear that as the number of connecting points in the base and top platforms was increased, the difficulties in deriving a single polynomial were magnified and its degree increased significantly (see Lin et al., 1992; Lin et al., 1994; Innocenti and Parenti-Castelli, 1993a, 1993b; Innocenti, 1995; Husty, 1996). It finally emerged that the polynomial for the general 6-6 platform was 40th degree.

A dilemma existed. Whereas the 3-3 octahedral platform was the most attractive because it is the most geometrically stable platform, having all triangular faces, and because its forward analysis requires the solution of only an 8th-degree polynomial, the design of concentric ball-and-socket joints is difficult and can lead to mechanical interference. This is unacceptable. On the other hand, a 6-6 platform avoids mechanical interference in design. However, a forward analysis requires the solution of a 40th-degree polynomial, which is impractical, and not geometrically stable if one deviates far from the octahedral platform. In other words, the center point of a pair of concentric ball-and-socket joints should be separated into a pair of distinct ball-and-socket joints whose centers are just far enough apart to avoid interference of the legs. Furthermore, if one separates each pair of concentric ball-and-socket joints along the sides of a triangle, a special 6-6 platform is obtained (see Figure 5) whose forward analysis requires the solution of only an 8th-degree polynomial. This design was patented by Griffis and Duffy (1993) through the University of Florida.

Since the publication of Fichter's (1986) cornerstone paper, the subject of parallel manipulators has attracted



In-Parallel Manipulator | Platforms with Compliant Legs | Tensegrity Structures with Ties | Self-Deployable Tensegrity Structures with Elastic Ties

(a) (b) (c) (d)

Figure 4. Evolution of Self-Deployable Structures

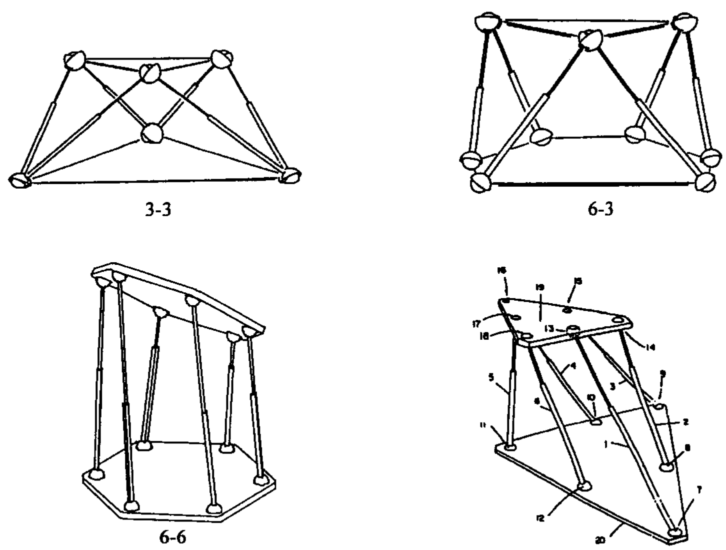


Figure 5. In-Parallel Devices

worldwide attention in academia accompanied by a multitude of papers published in journals and conferences. Because of space limitations, only a selection of relevant references are listed: Behi (1988), Chou and Sadler (1993), Haug et al. (1992), Masory (1993), Hunt and McAree (1998), Merlet (1987, 1988, 1989, 1994, 1995), Sugimoto (1989), and Zhuang et al. (1995).

2.2. Platforms with Compliant Legs

In 1988, Duffy and colleagues[1] began to study platform devices with compliant legs as illustrated in Figure 4b. Linear springs were inserted into each of the S-P-S connectors. Prior to this study, platforms had been used exclusively as flight simulators or as rides at amusement parks. In these applications, the top platform is free to pitch, roll, and yaw, and there is no contact with the environment.

The study of platforms with compliant legs was completely novel. Unlike previous applications, the top platform was to come into contact with the environment and be used to measure and control contact force. It is well known that a small change in force, δf, can be produced by a small displacement, δx, of a linear spring, and that $\delta f = k\delta x$, where k is the spring constant. Here, k can be considered as a stiffness-mapping matrix. Consider now that the preloaded compliant mechanism shown in Figure 4b is in equilibrium with an externally applied wrench (force--couple combination), $\hat{\mathbf{w}} = \begin{bmatrix} \mathbf{f} \\ \mathbf{m}_0 \end{bmatrix}$. Here, the force $\mathbf{f} = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$ and the couple $\mathbf{m}_0 = m_{0x}\mathbf{i} + m_{0y}\mathbf{j} + m_{0z}\mathbf{k}$ are a pair of vectors each in R_3. Consider now that there is a small change in the externally applied wrench, $\delta\hat{\mathbf{w}}$. The top platform will begin to twist on an infinitesimal screw, and the twist may be written as $\delta\hat{\mathbf{D}} = \begin{bmatrix} \delta\mathbf{S}_0 \\ \delta\boldsymbol{\varphi} \end{bmatrix}$. Here, the small displacement of a point in the top platform coincident with a reference point O is $\delta\mathbf{S}_0 = \delta x_0\mathbf{i} + \delta y_0\mathbf{j} + \delta z_0\mathbf{k}$ and the small rotation about the screw axis is $\delta\boldsymbol{\varphi} = \delta\varphi_x\mathbf{i} + \delta\varphi_y\mathbf{j} + \delta\varphi_z\mathbf{k}$. $\delta\mathbf{S}_0$ and $\delta\boldsymbol{\varphi}$ are also a pair of vectors in R_3.

A first objective of the study was to derive the 6×6 stiffness mapping (**K**) for the compliant mechanism such that $\partial\hat{\mathbf{w}} = [\mathbf{K}]\partial\hat{\mathbf{D}}$. This mapping was derived properly using line geometry and basic statics. The properties of the stiffness mapping were investigated by Griffis and Duffy (1990, 1991). A different but related study was performed by Loncaric (1985).

2.3. A Family of Self-Deploying Tensegrity Structures

It is important to recognize that we are not claiming any advanced expertise in the field of tensegrity structures in general. It is equally important to recognize that we possess an expertise in the analysis of in-parallel platforms with and without compliant legs or connectors. Such analyses are best based on screw theory and line geometry.

Figure 6 illustrates the family of tensegrity prisms under consideration. These are certainly not new. (However, we claim that the introduction of elastic ties to effect self-deployment is novel.) Fuller (1960) reported the octahedron tensegrity structure with the triangular base and top. As far as we are aware, the remainder of this family tabulated in the second column was first investigated by Tobie (1967). More recent accounts of these structures, which are called skew prisms, are presented by Kenner (1976) and Gabriel (1997). As pointed out by Tobie, each skew prism is derivable from a corresponding prism of parallel struts simply by an appropriate rotation α of the top platform relative to the base. The tensegrity pyramids in the third column are derived from the second column simply by reducing the upper faces in size. In practice, the struts have finite diameters, and as the upper face is reduced they will eventually touch and the structure is no longer a pyramid structure. However, models demonstrate that such structures will self-deploy from a stowed position.

A relative rotation angle α between each top and base can be measured by rotating the top of each prism in the first column, which is called a parallel prism because all the ties are parallel. The top can be rotated counterclockwise or clockwise, defining right- and left-handed tensegrity prisms, which are mirror images of each other. The tensegrity prisms in the second column are all counterclockwise. It is a remarkable result that for inelastic ties, the value for α is unique for each tensegrity prism and is given by $\alpha = 90 - \frac{180}{n}$, where n is the number of sides of the upper or lower polygons and, hence, α = 30, 45, 54, and 60 degrees for the triangle, square, pentagon, and hexagon, respectively. This result was derived by Kenner (1976) and is based on Tobie (1967). The value for α, together with the sizes of the tops and bases and their distance apart, enables one to compute the lengths of the struts and ties.

3. Deployable Tensegrity Papers

During the 1990s, tensegrity structures became increasingly applicable in space structure design, including space frames, precision mechanisms, and deployables. One of the leading researchers in this field (Motro, 1992) edited a special edition of the *International Journal of Space Structures* that was dedicated to tensegrity. Snelson (available at http://www.teleport.com/'pdx4d/docs/rmoto.html) wrote an introductory letter for this edition describing his invention, Fuller's contribution to its development, and the synergy between art and engineering. Motro's work has focused predominantly on the stability of tensegrity structures, including force density (Motro et al., 1994), nonlinear analysis (Kebiche et al., 1999), and morphology (Motro, 1996). Despite his clear focus on the engineering aspects of tensegrity, Motro has an excellent grasp of the artistic applications for this work. There is a clear development of stable, strut/tie structures from rectilinear (one-dimensional), to planar (two-dimensional), to spatial (three-dimensional). The 3-3, octahedron tensegrity is an excellent example of a spatial structure. Motro has developed multiple tensegrity structure designs that solve some of the toughest curved-surface problems for space structures. This class of structures is extremely lightweight, with excellent geometric stability and deployability.

The Motro (1996) paper is perhaps the best comprehensive review on the origins and applications of tensegrity. Motro explained the utility of tensegrity design in mechanical systems in very clear terms and with excellent reference to system requirements. A series of patents were issued between 1959 and 1965 by Snelson and Fuller that describe the mechanics and application to structural engineering. Motro states: "Tensegrity systems are composed of two sets of elements, a continuous set of cables, and a discontinuous set of rectilinear struts. The whole defines a reticulated space structure in a state of self-stress such as tension which is exclusively carried by cables and compression by struts" (p. 236). Based on this definition, the tensegrity application in self-deploying structures follows the assumption that all tensile members (ties) are in tension and all compression members (struts) are in compression. This can only be achieved when the structure is in a few configurations (tensegrity positions).

Wang (1998a, 1998b) has performed some of the best work on cable-strut systems as an extension of tensegrity. Reciprocal prisms (RP) and crystal-cell pyramidal (CP) grids, which technically exclude tensegrity systems, are the basis for his space frame applications. He developed a hierarchy of feasible cable-strut systems that include his new discoveries and tensegrity. Starting with triangular RP

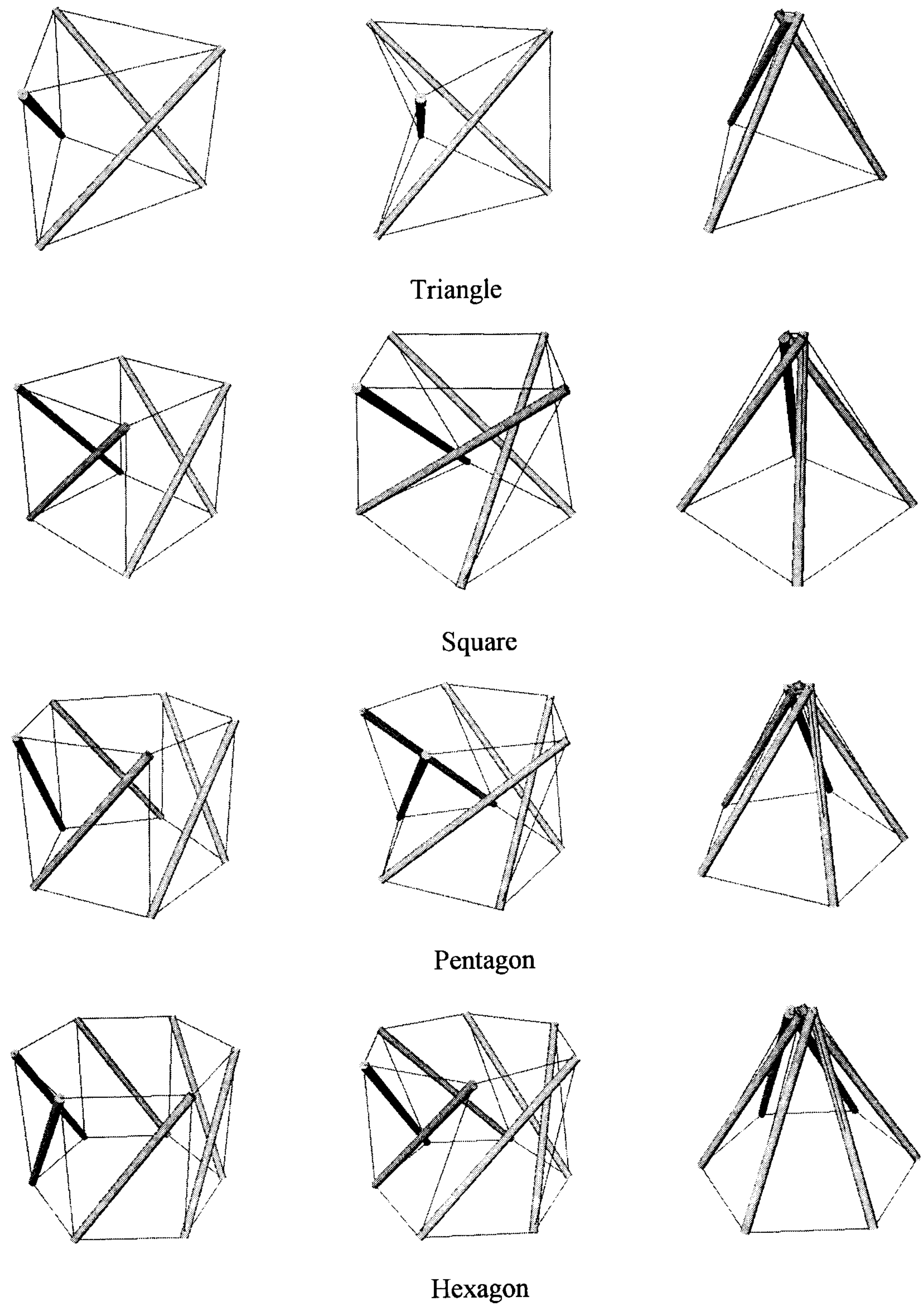


Figure 6. Family of Tensegrity Prisms

and CP simplexes, square, pentagonal, and hexangular systems are developed to build cable domes, ring beams (Wang, 1998c), and double-layer tensegrity grids (Wang and Liu, 1996). His work on the feasibility of these new applications is very important for space structure development.

Pellegrino and colleagues have focused on the application of tensegrity in deployable space structures. Precision is of great concern with these kinematic systems, and recent system developments have required even higher precision from much lighter structures. By developing the mathematics for cable-constrained nodes, You (1997) has been able to very accurately model the position of mesh antenna surfaces, including proven experimental results. Studies on the analysis of mechanisms (Calladine and Pellegrino, 1991), folding concepts for flexible but solid surface reflectors (Tibbalds et al., 1998), and shape control based on stress analysis (Kawaguchi et al., 1996) have all greatly contributed to the state of the art. Infinitesimal mechanism analysis has led to prestressing conditions that are critical for understanding deployable tensegrity structures. Pellegrino and colleagues' work with semisolid antenna reflectors has solved some of the fundamental problems associated with deploying these delicate systems. Launch capacity (size and weight) has continually reduced in recent years, requiring multiple folding systems to provide larger and larger structures. Obviously, once these structures are deployed and in operation, the surface must be maintained to meet performance requirements. Pellegrino and colleagues have led the community in predictive models for using stress profiles (and node position control) to ensure reflector surface positioning is maintained.

Skelton has seen the control of tensegrity structures as a new class of smart structures (Skelton and Sultan, 1997). This work has been applied to deployable telescope design (Sultan et al., 1999a), where precision is orders of magnitude tougher than deployable antennas. Skelton has also been instrumental in the development of integrated design (Sultan and Skelton, 1997) and reduction of prestress (Sultan et al., 1999b), which are critical for solving position correction and dynamic control issues.

Clearly, the applications for tensegrity structures have been continually studied since its creation by Snelson in 1949. Some applications (antennas) have been considered by some of the world's leading structural researchers. What sets our work apart from this previous work is the direct application to self-deployment. By using our background in kinematics and mathematics, the stability (and therefore utility) of these structures is assured. In the case of deployable space antennas, there is much expense and risk attributed to the deployment mechanism, which converts a stowed columnar structure into a parabolic reflective surface in orbit. Similarly, the deployable tent application provides a complete, single-component, stowed structure that can be easily self-deployed for field use. This allows for no specific tooling or potential for loss of system components. Deployable tensegrity structures are unique due to the significant reduction in systems complexity. The previous development of similar systems has been extremely difficult and risky, greatly increasing cost.

4. Closing Remarks

The architectural design community was instrumental in the development of tensegrity since its inception in the 1950s. This comprehensive work has benefited the engineering community by providing a multitude of concepts that have helped solve several structural problems. Clearly, Fuller and Snelson were the creative forces in the conception of tensegrity. The detailed work of numerous engineering groups in France, China, the United Kingdom, and the United States allows this practical application.

We have built on these significant accomplishments, adding kinematic structure and deployable space systems experience to define a new class of self-deploying structures using elastic ties.

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Note

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