Static Analysis of Prestressed Tensegrity Structures

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Abstract

In this paper the mathematical model to perform the static analysis of a prestressed antiprism tensegrity structure subjected to an arbitrary reduction of its connecting ties is addressed. A virtual work approach is used to deduce the equilibrium equations and the numerical results are verified using a Newtonian approach. One example is provided to illustrate the mathematical model.

1. Introduction

Tensegrity structures are spatial structures formed by a combination of rigid elements (the struts) and elastic elements (the ties). No pair of struts touch and the end of each strut is connected to three non-coplanar ties [1]. The struts are always in compression and the ties in tension. The entire configuration stands by itself and maintains its form solely because of the internal arrangement of the ties and the struts [2]. Tensegrity is an abbreviation of tension and integrity.

The development of tensegrity structures is relatively new and the works related have only existed for the 25 years. Kenner [3] established the relation between the rotation of the top and bottom ties. Tobie [2] presented procedures for the generation of tensile structures by physical and graphical means. Yin [1] obtained Kenner's results using energy considerations and found the equilibrium position for the unloaded tensegrity prisms. Stern [4] developed generic design equations to find the lengths of the struts and elastic ties needed to create a desired geometry. Since no external forces are considered his results are referred to the unloaded position of the structure. Knight [5] addressed the problem of stability of tensegrity structures for the design of a deployable antenna. Most recently Correa [6] obtained the equilibrium position for a general antiprism tensegrity structure subjected to a wide variety of external loads using virtual work. This method is used here.

It is said that a structure is prestressed when the free length of one or several of the connecting ties is decreased. In this paper the problem of the determination of the equilibrium position of a prestressed antiprism tensegrity structure is addressed. Also a software able to generate and solve the equations necessary to model the structure was developed in Matlab.

2. Nomenclature

Figure 1a shows a tensegrity structure formed by *n* struts each one of length L_S . In every structure it is possible to identify the top ties, the bottom ties and the lateral or connecting ties which are denoted as *T, B* and *L* respectively. Figure 1b shows the same structure. The bottom ends of each strut is labeled consecutively as $E_1, E_2, \cdots, E_j, \cdots, E_n$ where 1 identifies the first strut and *n* stands for the last strut. Similarly the top ends of the struts are labeled as $A_1, A_2, \cdots, A_j, \cdots, A_n$. The selection of the first strut is arbitrary but once it is chosen it should not be changed.

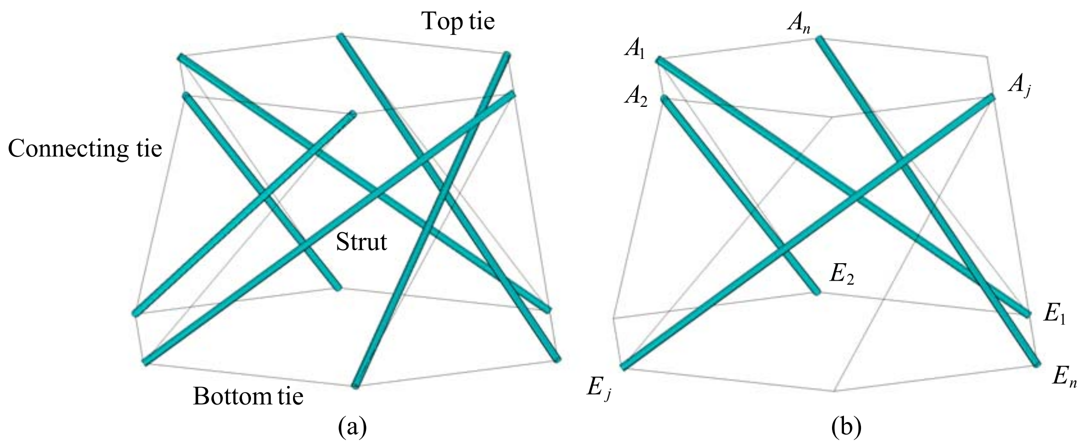


Figure 1. Nomenclature for tensegrity structures.
a) Components; b) Strut ends.

3. Generalized Coordinates and Transformations Matrices

Figure 2a shows an arbitrary point P located on a strut of length L_s. In a reference system D whose z axis is along the axis of the strut and with its origin located at the lower end of the strut, the coordinates of ${}^{D}\underline{P}$ are simply $(0,0,l)$. However frequently is more convenient for purposes of analysis to express the location of $\underline{P}$ in the global reference system A.

If the lower end of one strut is constrained to move on the horizontal plane and also the rotation about its longitudinal axis is constrained, the strut can be modeled by a universal joint. In this way the joint provides the 4 degrees of freedom associated with the strut. The total system has $4*n$ degrees of freedom which means there are $4*n$ generalized coordinates.

For each strut the generalized coordinates are the horizontal displacements a_j, b_j of the lower end of the strut together with two rotations about the axes of the universal joint, ε_j and β_j. ε_j corresponds to the rotation of the strut about ${}^{B}\underline{x}$ axis and β_j corresponds to the rotation about ${}^{C}\underline{y}$ axis, see Figure 2b.

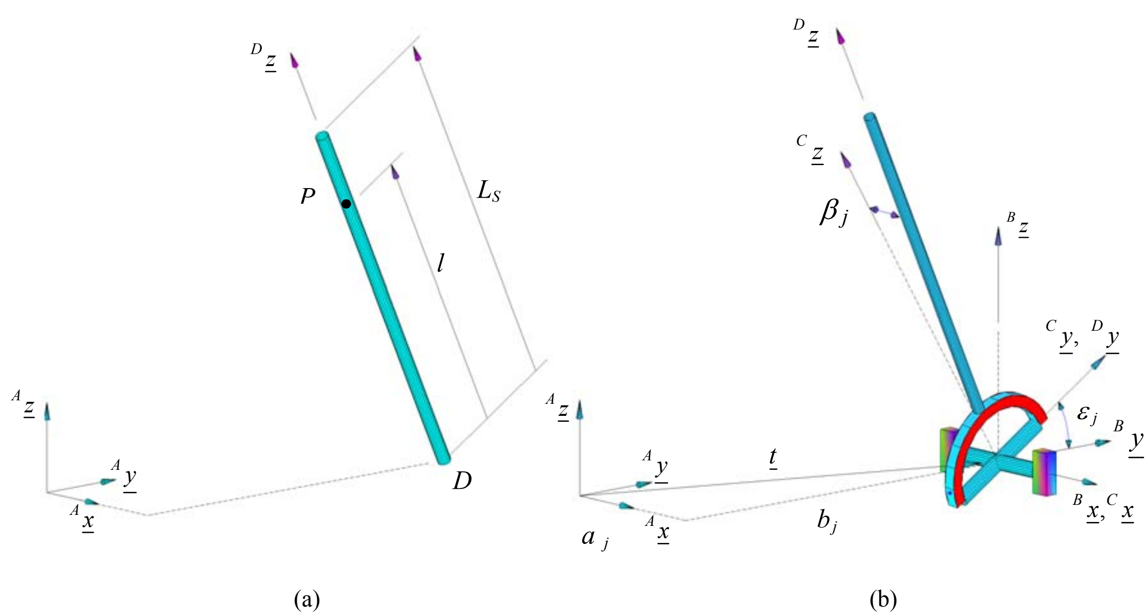


Figure 2. Degrees of freedom associated with one of the struts of a tensegrity structure.
a) Arbitrary point on the strut; b) Strut modeled as a universal joint.

The alignment of the z axis on the fixed system with the axis of the rod can be accomplished using the following three consecutive transformations, [7]: translation, $\underline{t}=(aj, bj, 0)$, rotation ε about the current x axis (${}^{B}\underline{x}$) and rotation β about the current y axis (${}^{C}\underline{y}$).

The coordinates of P expressed in the global reference system are

$$ {}^{A}\underline{P} = {}^{A}_{B}T_{a,b,0}\ {}^{B}_{C}T_{\varepsilon}\ {}^{C}_{D}T_{\beta}\ {}^{D}\underline{P} \tag{1} $$

where

$$ {}^{A}_{B}T_{a,b,0} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ {}^{B}_{C}T_{\varepsilon} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon & 0 \\ 0 & \sin\varepsilon & \cos\varepsilon & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

$$ {}^{C}_{D}T_{\beta} = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

$$ {}^{D}\underline{P} = [0 \ \ 0 \ \ l \ \ 1]^{T} \tag{5} $$

Substituting the above three expressions into (1) yields

$$ {}^{A}\underline{P} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} l\sin\beta + a \\ -l\sin\varepsilon\cos\beta + b \\ l\cos\varepsilon\cos\beta \\ 1 \end{bmatrix} \tag{6} $$

In addition to the constraint imposed that the lower ends are to remain in the horizontal plane and for each strut to avoid the rotation about its longitudinal axis the following assumptions are made without loss of generality:

- The struts are massless.
- All the struts have the same length.
- There are no dissipative forces acting on the system.
- The free lengths of the top ties are equal.
- The free lengths of the bottom ties are equal.
- The free lengths of the connecting ties are the same at the unstressed position.
- The stiffness of all the top ties is the same.
- The stiffness of all the bottom ties is the same.
- The stiffness of all the connecting ties is the same.

4. Coordinates of The Ends of the Struts

The Cartesian coordinates of the lower ends $\underline{E}_j$, expressed in the global reference system A, are obtained in terms of the generalized coordinates substituting l in (6) by 0, then

$$ {}^{A}\underline{E}_j = \begin{bmatrix} a_j \\ b_j \\ 0 \end{bmatrix} \tag{7} $$

Similarly the coordinates of the upper end of the struts A_j are evaluated substituting l by the length of the struts L_S in (6)

$$^{A}\underline{A}_j = \begin{bmatrix} L_s \sin\beta_j + a_j \\ -L_s \sin\varepsilon_j \cos\beta_j + b_j \\ L_s \cos\varepsilon_j \cos\beta_j \end{bmatrix} \quad (8)$$

Equations (7) and (8) permit one to obtain expressions for the lengths of the top, bottom and lateral ties in terms of the generalized coordinates as follows

$$T_j = \left(\left(A_{j+1,x} - A_{j,x}\right)^2 + \left(A_{j+1,y} - A_{j,y}\right)^2 + \left(A_{j+1,z} - A_{j,z}\right)^2\right)^{1/2} \quad (9)$$

$$B_j = \left(\left(E_{j+1,x} - E_{j,x}\right)^2 + \left(E_{j+1,y} - E_{j,y}\right)^2 + \left(E_{j+1,z} - E_{j,z}\right)^2\right)^{1/2} \quad (10)$$

$$L_j = \left(\left(A_{j,x} - E_{j+1,x}\right)^2 + \left(A_{j,y} - E_{j+1,y}\right)^2 + \left(A_{j,z} - E_{j+1,z}\right)^2\right)^{1/2} \quad (11)$$

where if $j = n$ then $j+1 = 1$

5. The Principle of Virtual Work for Tensegrity Structures

The virtual work for systems able to store potential energy can be stated from [8]

$$\delta W = \delta W_F + \delta W_M - \delta V \quad (12)$$

where δW is the total virtual work, δW_F is the total virtual work performed by non-conservative forces, δW_M is the total virtual work performed by non-conservative moments and δV is the differential of the potential energy.

The principle of virtual work states that in the equilibrium (12) vanishes. In addition since there are no external loads (12) can be rewritten as

$$\delta V = 0 \quad (13)$$

8. The Potential Energy

Since the struts are considered massless the term related to the potential energy in the principle of virtual work is the resultant of the elastic potential energy contributions given by the ties. The potential elastic energy for a general tie j is given by, [8]

$$V_j = \frac{1}{2} k \ (w_j - w_{j0})^2 \quad (14)$$

where V_j is the elastic potential energy for tie j, k the tie stiffness, w_j the current length of the tie j and w_{j0} the free length of the tie j. Therefore the differential of the potential energy for tie j is

$$\delta V_j = k \ (w_j - w_{j0}) \ \delta w_j \quad (15)$$

The differential of the potential energy for all the tensegrity structure, δV , is the resultant of the contributions of the top ties, the bottom ties and the lateral ties and can be expressed as

$$\delta V = \sum_{j=1}^{n} k_T\left(T_j - T_o\right)\delta T_j + \sum_{j=1}^{n} k_B\left(B_j - B_o\right)\delta B_j + \sum_{j=1}^{n} k_L\left(L_j - L_{oj}\right)\delta L_j \quad (16)$$

where k_T, k_B, k_L are the stiffness of the top, bottom and connecting ties respectively, T_j, B_j and L_j are given by (9), (10) and (11) and are functions of some of the generalized coordinates and T_0, B_0 and L_0 are the free lengths of the top, bottom and connecting ties respectively. It should be noted since the prestressing is obtained reducing the free lengths of the connecting ties, the values of L_0 may change for each connecting tie.

9. The General Equations

Now that each one of the terms contributing to the virtual work has been evaluated, the equilibrium condition for the general tensegrity structure can be established. Substituting (16) into (13) and re-grouping yields

$$f_1\delta a_1 + f_2\delta a_2 + \ldots\ldots f_n\delta a_n + f_{n+1}\delta b_1 + f_{n+2}\delta b_2 + \ldots\ldots f_{2n}\delta b_n +$$
$$f_{2n+1}\delta\varepsilon_1 + f_{2n+2}\delta\varepsilon_2 + \ldots\ldots f_{3n}\delta\varepsilon_n + f_{3n+1}\delta\beta_1 + f_{3n+2}\delta\beta_2 + \ldots\ldots f_{4n}\delta\beta_n = 0 \tag{17}$$

where

$$f_i = \sum_{j=1}^{n} k_T \left(T_j - T_o\right)\frac{\partial T_j}{\partial a_i} + \sum_{j=1}^{n} k_B \left(B_j - B_o\right)\frac{\partial B_j}{\partial a_i} + \sum_{j=1}^{n} k_L \left(L_j - L_{oj}\right)\frac{\partial L_j}{\partial a_i} \tag{18}$$

$$f_{n+i} = \sum_{j=1}^{n} k_T \left(T_j - T_o\right)\frac{\partial T_j}{\partial b_i} + \sum_{j=1}^{n} k_B \left(B_j - B_o\right)\frac{\partial B_j}{\partial b_i} + \sum_{j=1}^{n} k_L \left(L_j - L_{oj}\right)\frac{\partial L_j}{\partial b_i} \tag{19}$$

$$f_{2n+i} = \sum_{j=1}^{n} k_T \left(T_j - T_o\right)\frac{\partial T_j}{\partial \varepsilon_i} + \sum_{j=1}^{n} k_B \left(B_j - B_o\right)\frac{\partial B_j}{\partial \varepsilon_i} + \sum_{j=1}^{n} k_L \left(L_j - L_{oj}\right)\frac{\partial L_j}{\partial \varepsilon_i} \tag{20}$$

$$f_{3n+i} = \sum_{j=1}^{n} k_T \left(T_j - T_o\right)\frac{\partial T_j}{\partial \beta_i} + \sum_{j=1}^{n} k_B \left(B_j - B_o\right)\frac{\partial B_j}{\partial \beta_i} + \sum_{j=1}^{n} k_L \left(L_j - L_{oj}\right)\frac{\partial L_j}{\partial \beta_i} \tag{21}$$

$i = 1,\ 2,\ .\ .\ .\ ,\ n$

Equation (17) must be satisfied for all the values of the virtual displacements which in general are different from zero, then

$$\begin{aligned} f_1 &= 0 \\ f_2 &= 0 \\ &\vdots \\ f_{4n} &= 0 \end{aligned} \tag{22}$$

where f_i is given by equations (18) to (21). Equations (22) represent a strongly coupled system of $4{*}n$ equations depending only on the $4{*}n$ generalized coordinates. The equilibrium position for a general tensegrity structure is obtained by solving numerically the set (22) for a_1, b_1, ε_1, β_1, ... , a_n, b_n, ε_n, β_n. After that equations (7) and (8) yield explicitly expressions for the coordinates of the ends of the struts in the global coordinate system.

10. Initial Conditions

To be able to solve (22) it is necessary to find a proper set of values for the generalized coordinates in the unloaded position. This is accomplished using Yin's method [1], which is presented here without demonstration. In this method the three unknowns R_B, R_T and the length of the connecting ties L in equations (23), (24) and (25) are solved

$$k_L \left(1 - \frac{L_o}{L}\right) R_B - 2k_T \left(R_T - R_{To}\right) \sin\frac{\gamma}{2} = 0 \tag{23}$$

$$k_L \left(1 - \frac{L_o}{L}\right) R_T - 2k_B \left(R_B - R_{Bo}\right) \sin\frac{\gamma}{2} = 0 \tag{24}$$

$$L - \sqrt{L_s^2 + 2R_B R_T \left[\cos(\alpha + \gamma) - \cos\alpha\right]} = 0 \tag{25}$$

where

$$R_{T0} = \frac{T_o}{2\sin\frac{\gamma}{2}} \quad \text{and} \quad R_{B0} = \frac{B_o}{2\sin\frac{\gamma}{2}} \tag{26}$$

And the angles γ and α are given by

$$\gamma = \frac{2\pi}{n} \text{ and } \alpha = \frac{\pi}{2} - \frac{\pi}{n} \tag{27}$$

where n is the number of struts.

The values of R_B and R_T are then substituted into the equations (28) through (32) which yield the generalized coordinates for the unloaded position.

$$a_{j,0} = R_B \cos\left((j-1)\ \gamma \right) \ ,\ j = 1,\ 2,\ ....\ ,\ n \tag{28}$$

$$b_{j,0} = R_B \sin\left((j-1)\ \gamma \right) \ ,\ j = 1,\ 2,\ ....\ ,\ n \tag{29}$$

$$\tan \varepsilon_{j,0} = \frac{b_{j,0} - R_T \sin(\ (j-1)\ \ \gamma + \alpha)}{H} \tag{30}$$

$$\tan \beta_{j,0} = \frac{R_T \cos(\ (j-1)\ \ \gamma + \alpha) - a_{j,0}}{\left(\dfrac{b_{j,0} - R_T \sin(\ (j-1)\ \ \gamma + \alpha)}{\sin \varepsilon_{j,0}} \right)} \tag{31}$$

$$\text{where } H = \sqrt{L_s^2 - R_B^2 - R_T^2 - 2 R_B R_T \sin\frac{\gamma}{2}} \tag{32}$$

and if $j = 1$ then $j - 1 = n$

11. Verification of the Numerical Results

Because of the complexity of the equilibrium equations it is essential to verify the answers so obtained. An independent validation of the results can be accomplished isolating an strut and performing the summation of moments with respect to the lower end of the strut, see Figure 3

$$\underline{r} \times \underline{F}_{Aj\ Aj+1} + \underline{r} \times \underline{F}_{Aj\ Aj-1} + \underline{r} \times \underline{F}_{Aj\ Ej+1} = \underline{0} \tag{33}$$

where

$$\underline{r} = \underline{A}_j - \underline{E}_j \tag{34}$$

$$\underline{F}_{Aj\,Aj+1} = k_T \left(\left| \underline{A}_{j+1} - \underline{A}_j \right| - T_0 \right) \frac{\underline{A}_{j+1} - \underline{A}_j}{\left| \underline{A}_{j+1} - \underline{A}_j \right|} \tag{35}$$

$$\underline{F}_{Aj\,Aj-1} = k_T \left(\left| \underline{A}_{j-1} - \underline{A}_j \right| - T_0 \right) \frac{\underline{A}_{j-1} - \underline{A}_j}{\left| \underline{A}_{j-1} - \underline{A}_j \right|} \tag{36}$$

$$\underline{F}_{Aj\,Ej+1} = k_L \left(\left| \underline{E}_{j+1} - \underline{A}_j \right| - L_{0j} \right) \frac{\underline{E}_{j+1} - \underline{A}_j}{\left| \underline{E}_{j+1} - \underline{A}_j \right|} \tag{37}$$

If after substituting the numerical values and the evaluation of (33) is the zero vector $\underline{0}$ then the current position is an equilibrium position.

12. Example: Analysis of a Tensegrity Structure with 5 Struts

12.1 ANALYSIS FOR THE UNPRESTRESSED POSITION

It is required to evaluate the unprestressed equilibrium position of a tensegrity structure with 5 struts and with the stiffness and free lengths shown in Table 1. Each of the struts has a length $L_s = 65mm$.

Table 1. Stiffness and free lengths for the structure.

	Stiffness (N/mm)	Free lengths (mm)
Top ties	$k_T = 0.5$	$T_0 = 35$
Bottom ties	$k_B = 0.3$	$B_0 = 45$
Connecting ties	$k_L = 0.4$	$L_{0,initial} = 65$

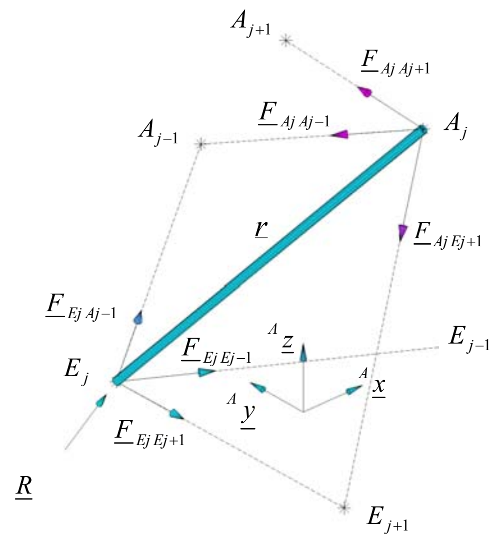


Figure 3. Free body diagram of an arbitrary strut.

For this example $n = 5$ then (26) and (27) yield $\gamma = 72°$, $\alpha = 54°$, $R_{T0} = 29.7728mm$ and $R_{B0} = 38.2793mm$.

The solution of (23), (24) and (25) yields $R_B = 41.2035mm$, $R_T = 32.0298mm$ and from (32), $H = 84.1287mm$. The initial values of the generalized coordinates are obtained from (27) through (31) and are listed in Table 2.

Table 2. Initial values of the generalized coordinates for the structure.

	Strut 1	Strut 2	Strut 3	Strut 4	Strut 5
a (mm)	41.2035	12.7326	-33.3343	-33.3343	12.7326
b (mm)	0	39.1869	24.2188	-24.2188	-39.1869
ε (rad)	-0.3968	0.6709	0.7380	-0.2275	-0.8110
β (rad)	-0.7302	-0.5006	0.3794	0.7879	0.0678

Using the values of Table 2, equations (7) and (8) yield the coordinate of the ends of the structure for the unloaded position. The results are illustrated in Table 4 and Figure 4a.

12.2 ANALYSIS FOR THE PRESTRESSED POSITION

It is required to evaluate the final equilibrium position of the structure when the free lengths of the connecting ties attached to the upper ends A_1 and A_5 are decreased to 35 mm and the horizontal displacements of the lower ends E_1 and E_2 are constrained, see Figure 4a.

Since the system has 5 struts and there are 4 constraints (a_1, b_1, a_2 and b_2) then there are 16 degrees of freedom and therefore 16 equations are required, one per each generalized coordinate. Equation (18) yields f_3, f_4 and f_5, equation (19) yields f_8, f_9 and f_{10}, equation (20) yields f_{11}, f_{12}, f_{13}, f_{14} and f_{15} and equation (21) yields f_{16}, f_{17}, f_{18}, f_{19} and f_{20}. It should be noted that in (21) $L_{01} = L_{05} = 35mm$ and $L_{02} = L_{03} = L_{04} = 65mm$ Each f_i is equated to zero and then the system is solved numerically using the software developed and the initial values listed in Table 2. The values of the parameters that determine the final position are listed in Table 3. Note that the values of a_1, b_1, a_2 and b_2 in Tables 2 and 3 do not change because they are constrained

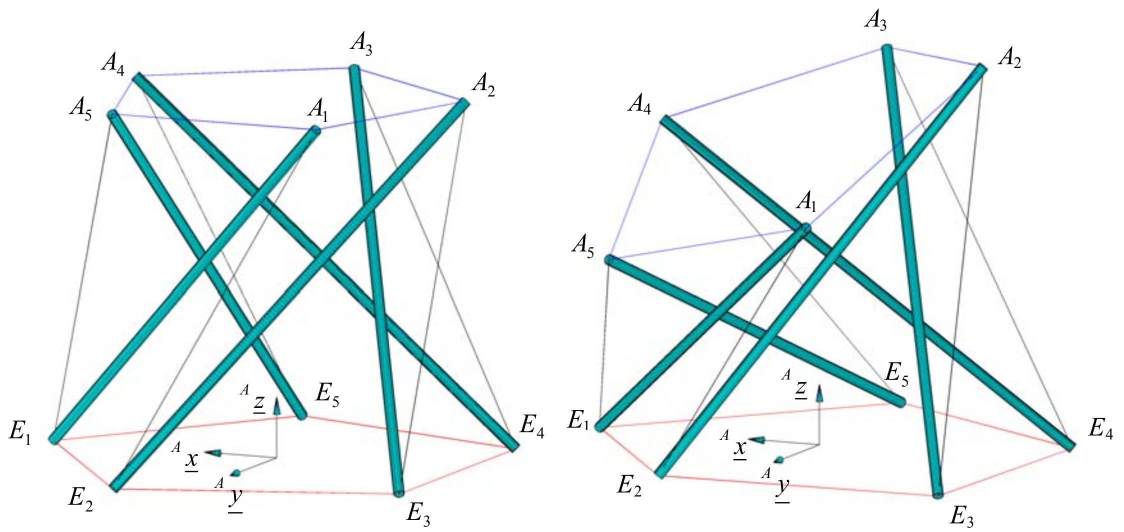
Table 3. Parameters for the final position for the structure.

	Strut 1	Strut 2	Strut 3	Strut 4	Strut 5
a (mm)	41.2035	12.7326	-39.1287	-41.0448	3.8991
b (mm)	0	39.1869	37.9301	-14.3338	-43.2610
ε (rad)	-0.8005	0.3169	0.6427	-0.2799	-1.1453
β (rad)	-0.7322	-0.5965	0.3644	0.8879	0.2143

Using the values of Tables 3, equations (7) and (8) yield the coordinates of the ends of the struts for the initial and final position. The results are presented in Table 4 and Figure 4b.

Table 4. Lower and upper coordinates for the unloaded and final position for the structure (mm).

	Strut 1		Strut 2		Strut 3		Strut 4		Strut 5	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final	Initial	Final
E_x	41.203	41.203	12.732	12.732	-33.33	-39.12	-33.33	-41.04	12.732	3.8991
E_y	0	0	39.186	39.186	24.218	37.930	-24.21	-14.33	-39.18	-43.26
E_z	0	0	0	0	0	0	0	0	0	0
A_x	-18.82	-18.96	-30.46	-37.82	0	-7.052	30.462	28.771	18.826	23.039
A_y	25.912	48.039	-9.897	15.986	-32.02	-12.46	-9.897	1.3579	25.912	36.838
A_z	61.845	46.609	61.845	70.750	61.845	67.314	61.845	54.585	61.845	36.300



(a) (b)

Figure 4. Unprestressed and final positions for the structure.

Figure 5 shows the free body diagram for the second strut in its final position. Equation (33) for the present strut has the form

$$\underline{r} \times \underline{F}_{A2A1} + \underline{r} \times \underline{F}_{A2A3} + \underline{r} \times \underline{F}_{A2E3} = \underline{0} \quad (38)$$

With the aid of the initial values given in Table 1, the coordinates for the final position listed in Table 4 and recalling that the free length of the connecting tie L_{02} remains unchanged in 65 mm, equations (34) through (37) yield

$$\underline{r} = \underline{A}_2 - \underline{E}_2 \quad = \begin{bmatrix} -50.5579 \\ -23.2003 \\ 70.7506 \end{bmatrix} mm$$

$$\underline{F}_{A2A1} = k_T(|\underline{A}_1 - \underline{A}_2| - T_0)\frac{\underline{A}_1 - \underline{A}_2}{|\underline{A}_1 - \underline{A}_2|} \quad = \begin{bmatrix} 1.9869 \\ 3.3760 \\ -2.5427 \end{bmatrix} N$$

$$\underline{F}_{A2A3} = k_T(|\underline{A}_3 - \underline{A}_2| - T_0)\frac{\underline{A}_3 - \underline{A}_2}{|\underline{A}_3 - \underline{A}_2|} \quad = \begin{bmatrix} 2.5804 \\ -2.3860 \\ -0.2882 \end{bmatrix} N$$

$$\underline{F}_{A2E3} = k_L(|\underline{E}_3 - \underline{A}_2| - L_{02})\frac{\underline{E}_3 - \underline{A}_2}{|\underline{E}_3 - \underline{A}_2|} \quad = \begin{bmatrix} -0.0639 \\ 1.0766 \\ -3.4711 \end{bmatrix} N$$

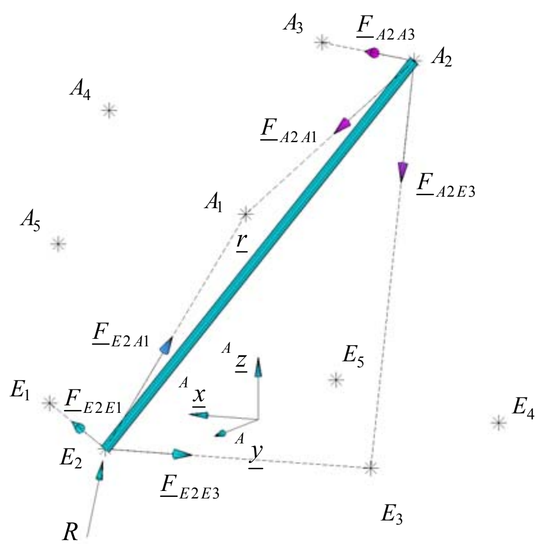


Figure 5. Free body diagram for the second strut in the final position.

After substituting the previous values in (38) the result is *0* which means the current position is an equilibrium position. The software does internally the same operation for each strut.

13. Conclusions

The model developed here allows one to analyze a general prestressed antiprism tensegrity structure subjected to a prestressing action.

The model is developed using the virtual work approach and all the results are checked using the Newton’s Third Law. This verification assures one that the answers produced by the numerical method accurately correspond to equilibrium positions.

The presentation of the results in graphic form aids to the understanding of the complex configuration that a structure can assume after an arbitrary prestressing of its connecting ties.

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