Editorial

The Fallacy of Modern Hybrid Control Theory that is Based on "Orthogonal Complements" of Twist and Wrench Spaces

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This editorial is intended to be a plea to researchers in the field of robotic systems to reconsider the theory of so-called "hybrid control" for the simultaneous control of force and motion.

The theory I refer to is one that has been used in quite a large number of articles that have been presented in both national and international conferences on robotics and in the technical journals of learned societies. In these articles the aims are to model constrained-motion tasks, grasping operations and calibration of robots using "orthogonal complements" in one form or another. Because the basis for these articles is fallacious, they can only be called erroneous except when, serendipitously, a correct result appears through the simplicity or symmetry of the problem; in such cases the articles become dangerously misleading.

It is of even greater concern to see the fallacious arguments enshrined in several textbooks on robotics for engineering students. These books contain chapters which explain in detail the meaning and the use of "orthogonal complements."

Reduced to simplest terms, the fallacy lies in the definition of "orthogonality" applied to instantaneous rigid-body motions, called twists and equally to forces/

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couples in combinations, these being called wrenches. Twists and wrenches are not new concoctions. They date back well over a century. The definition of orthogonality that generates the erroneous "orthogonal complements" in hybrid control, is, essentially, a statement that two twists

$$T_1 = [\mathbf{V}_1; \mathbf{\Omega}_1]$$

and

$$T_2 = [\mathbf{V}_2; \mathbf{\Omega}_2]$$

are orthogonal when

$$T_1 \cdot T_2 = \mathbf{V}_1 \cdot \mathbf{V}_2 + \mathbf{\Omega}_1 \cdot \mathbf{\Omega}_2 = 0^*.$$

Admittedly this statement does not usually appear in the literature in this form, but rather it appears in the form of sparse matrices containing ones and zeros, such as

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad S^{\perp} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is so plausible, so intuitive, and at first sight so good an idea to state that the pair of column vectors in S are "orthogonal" and to go further and to state that S and $S^{\perp}$ are "orthogonal complements." However the dimensions of elements of this matrix representation are not immediately obvious, whereas it is blatantly obvious that the dimensions for the products $\mathbf{V}_1 \cdot \mathbf{V}_2$ and $\mathbf{\Omega}_1 \cdot \mathbf{\Omega}_2$ are, respectively, (length/s)^2 and (radian/s)^2. The expression $T_1 \cdot T_2$ is therefore non-homogeneous. For example, employing this definition of orthogonality, the following pair of twists

$$T_1 = [1, 1, 1; 2, 2, 2]$$

and

$$T_2 = [2, 2, 2; -1, -1, -1],$$

*A pair of wrenches $w_1 = [\mathbf{f}_1; \mathbf{m}_1]$ and $w_2 = [\mathbf{f}_2; \mathbf{m}_2]$ are similarly said to be 'orthogonal' when $w_1 \cdot w_2 = \mathbf{f}_1 \cdot \mathbf{f}_2 + \mathbf{m}_1 \cdot \mathbf{m}_2 = 0$.

are "orthogonal" ($T_1 \cdot T_2 = 0$) when, for instance, $\mathbf{V}_1$ and $\mathbf{V}_2$ are measured in m/s. Consider now that $\mathbf{V}_1$ and $\mathbf{V}_2$ are to be measured in cm/s; then the *exact same pair of motions* are expressed by

$$T_1 = [1 \times 10^2, 1 \times 10^2, 1 \times 10^2; 2, 2, 2]$$

and

$$T_2 = [2 \times 10^2, 2 \times 10^2, 2 \times 10^2; -1, -1, -1].$$

It is obvious that the twists are no longer "orthogonal" because, $T_1 \cdot T_2 \neq 0$.

Moreover it is a simple matter to check that, given two instantaneous rigid-body motions for which "orthogonality" (as above defined) *is* satisfied, when we shift the origin of the coordinates used in the determination, the condition of "orthogonality" is no longer satisfied. Accordingly this kind of "orthogonality" is, from the practical point of view, meaningless, because it is origin-dependent. It can even be said that, given *any* two instantaneous rigid-body motions, it is possible to find points that can be used as origin of coordinates for which they *are* "orthogonal complements."And this conclusion is ridiculous.

Thus the absurdity is obvious if one accepts that one can obtain an ∞^3 of answers to the same problem depending upon which point one selects as the origin for a coordinate system (an ∞^3 of points fill space)! Can a robot manipulator really behave differently in an ∞^3 ways when performing a given constrained-motion task? The preposterous conclusion, that it can, follows from the dependence on the choice of the origin. For example, if an origin of a reference system for a simple insertion task is chosen off the axis of the hole, an unwanted and uncontrollable translational velocity parallel to the hole axis can be induced causing either an unacceptable insertion rate or a withdrawal of the peg.

Conclusion: *Modern hybrid control theory based on the definition of orthogonality, shown above to be fallacious, is completely devoid of meaning whether from the point of view of practicability or geometry. For, when we use such a theory to determine whether two given instantaneous motions are "orthogonal," we encounter* (1) *dimensional inconsistency*, (2) *dependence on the choice of units used*, *and* (3) *dependence on the choice of the origin of coordinates*.

Now let me justify even further my categorical debunking of the fallacious theory by viewing the geometry of twists (and wrenches) from a more advanced viewpoint that is well established in both geometry and mechanics. Twists and wrenches can be properly represented in terms of real orthogonal spaces (see for example [1, 2]). viz. a real orthogonal space is a real vector space endowed with a symmetric bilinear form. Consider that two twists are represented by

$$T = [V_{ox}, V_{oy}, V_{oz}; \Omega_x, \Omega_y, \Omega_z]$$

and

$$T^\star = [V^\star_{ox}, V^\star_{oy}, V^\star_{oz}; \Omega^\star_x, \Omega^\star_y, \Omega^\star_z].$$

The fallacious definition of orthogonality endows the space with the following bilinear form,

$$H(T, T^\star) = V_{ox}V^\star_{ox} + V_{oy}V^\star_{oy} + V_{oz}V^\star_{oz} + \Omega_x\Omega^\star_x + \Omega_y\Omega^\star_y + \Omega_z\Omega^\star_z,$$

The corresponding quadratic form is positive definite,

$$H(T, T) = V^2_{ox} + V^2_{oy} + V^2_{oz} + \Omega^2_x + \Omega^2_y + \Omega^2_z.$$

This again displays the dimensional inconsistancy, and therefore, it should not be used.

However for the group of Euclidean motions, the orthogonal space for twists is *in fact* endowed with two symmetric bilinear forms*, and linear combinations of them can be employed. *No other symmetric bilinear form can be meaningfully employed*. The first is

$$Kl(T, T^\star) = V_{ox}\Omega^\star_x + V_{oy}\Omega^\star_y + V_{oz}\Omega^\star_z + \Omega_x V^\star_{ox} + \Omega_y V^\star_{oy} + \Omega_z V^\star_{oz},$$

which is known as the Klein form. The corresponding quadratic form is

$$Kl(T, T) = 2(V_{ox}\Omega_x + V_{oy}\Omega_y + V_{oz}\Omega_z),$$

which is indefinite.† The second bilinear form is degenerate (the kernel of the associated vector space is non-zero),

$$Ki(T, T^\star) = (0)V_{ox}V^\star_{ox} + (0)V_{oy}V^\star_{oy} + (0)V_{oz}V^\star_{oz} + \Omega_x\Omega^\star_x + \Omega_y\Omega^\star_y + \Omega_z\Omega^\star_z$$

which is known as the Killing form (each (0) being a zero). The corresponding quadratic form is

$$Ki(T, T) = \Omega^2_x + \Omega^2_y + \Omega^2_z,$$

which is positive semidefinite. Analogous expressions can straight away be written for the orthogonal spaces of wrenches.

Both $Kl(T, T^\star)$ and $Ki(T, T^\star)$ are not only invariant with a choice of origin but also with a change of units, and a pair of twists can be orthogonal with respect to either or both of the Klein and the Killing forms. For instance, when two twists are orthogonal with respect to the Killing form the axes of their rotations are mutually perpendicular.

On the contrary the employment of the symmetric bilinear form $H(T, T^\star) = 0$ to define orthogonal subspaces and to define "natural constraints" as the "orthogonal complements" of artificial constraints lacks all meaning. *Using such*

*R. Von Mises recognized both of these symmetric bilinear forms back in 1924.

†It is zero when the twist is a pure rotor or a pure translation. Otherwise it is positive or negative.

a formulation the subspaces defining natural constraints are not invariant with a choice of origin or a change of dimensions. The nonsensical outcome of applying "orthogonal complements" in hybrid control leads to the definition of terms such as "natural constraints" or "motion a body cannot have," which are absolutely meaningless. The "*motion a body cannot have*" *does not even constitute a linear subspace*. It is not closed under addition.

Sir Robert Stawell Ball got it right, over a hundred years ago when he defined *twists of freedom* to quantify the constrained motion of a rigid body and *wrenches of constraint* to quantify the forces and couples which produce no motion of a quiescent body.[3,4] Any wrench of constraint w is related to any twist of freedom T by the vanishing of the reciprocal product of Ball,

$$w \cdot T = [\mathbf{f}; \mathbf{m}] \cdot [\mathbf{V}; \mathbf{\Omega}] = \mathbf{f} \cdot \mathbf{V} + \mathbf{m} \cdot \mathbf{\Omega}$$

which can be related directly to the vanishing of the Klein form $Kl(T, T^{\star}) = 0$.

My brief remarks support, I hope, my deep concern about the wrong track that quite a number of workers have taken in the field of hybrid control. I am the first to admit that Sugimoto and I were wrong about "orthogonality" in our joint article.[5] I deliberately refrain from drawing the reader's attention to any other specific articles that stem from the fallacious theory I have outlined, but anybody sufficiently interested can easily trace some of them. I confess that when I co-authored reference 5, the "orthogonal complement" did seem to be a good idea at the time! However, since then, a number of articles have been published which attempt to present a proper treatment of kinestatics by establishing a firm geometric foundation.[6–9]* These articles stem from reference 10.

The author sincerely hopes that this editorial has explained the issues in simple terms so that a reader can appreciate easily the problems which exist with the "orthogonal complements" associated with hybrid control. In addition to reference 10, three theses have appeared,[11–13] which further clarify the geometric meaning of twists and wrenches. The last thesis addresses a number of key experimental issues involved.

Finally, Loncaric and Brockett[14–16] explained that there "*is no natural positive definite metric on SE*(3)." This important result was obtained independently of the literature available on the subject that was published in engineering and robotic journals prior to 1985.

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*Reference 9 addresses the problem of origin dependence of hybrid control theory described earlier by applying hybrid control theory to an insertion task with the origin of the reference system chosen off the hole axis. Unwanted and uncontrollable translational velocities parallel to the hole axis are induced.

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