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ANALYSIS OF THREE DEGREE OF FREEDOM 6×6 TENSEGRITY PLATFORM

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ABSTRACT

The mechanism studied in this paper is a three degree of freedom 6×6 tensegrity structure. A tensegrity structure is one that balances internal (pre-stressed) forces of tension and compression. These structures have the unique property of stabilizing themselves if subjected to certain types of disturbances. The structure analyzed in this paper consists of two rigid bodies (platforms) connected by a total of six members. Three of the members are noncompliant constant-length struts and the other three members consist of springs. For typical parallel mechanisms, if the bottom platform is connected to the ground and the top platform is connected to the base by six compliant leg connectors, the top platform will have six degrees of freedom relative to the bottom platform. However, because three of the six members connecting the two platforms are noncompliant constant-length struts, the top platform has only three degrees of freedom.

The primary contribution of this paper is the analysis of the three degree of freedom tensegrity platform. Specifically, given the location of the connector points on the base and top platforms, the lengths of the three noncompliant constant-length struts, and the desired location of a point embedded in the top platform measured with respect to a coordinate system attached to the base, all possible orientations of the top platform are determined.

INTRODUCTION

The word *tensegrity* is a combination of the words *tension* and *integrity*. Tensegrity describes a structural relationship principle in which structural shape is guaranteed by the closed, continuous, tensional behaviors of the system and not by the discontinuous compressional member behaviors. Tensegrity provides the ability of a structure in theory to yield increasingly without ultimately breaking [Fuller,1979].

This paper will present an analysis of the geometric properties of platforms which incorporate tensegrity principles. A platform is described as any device that has multiple legs connecting a moving (top) platform to a bottom (base) platform [Abbasi, Ridgeway, Adsit, Crane, Duffy, 2000]. It was only recently that the forward position solution of a 3×3 platform was formulated by [Griffis and Duffy, 1989]. In their analysis, all positions and orientations of the top platform are determined based on given lengths of the six leg connectors. This 3×3 platform (Figure 1) was the simplest of the geometries to solve. The formulation of this solution yielded and eighth degree polynomial in the square of one defining parameter. Later, this solution technique was applied to a 6×3 platform (Figure 2). A forward displacement analysis of a general 6×6 constant strut length platform (see Figure 3) showed that the solution was in the form of a 40^{th} degree polynomial [Raghavan, 1993].

The tensegrity structure analyzed in this paper consists of a special 6×6 platform. This geometry was designed to include the benefits of a 3×3 platform and a general 6×6 platform [Griffis and Duffy, 1993]. This special platform makes the analysis comparable to a 3×3 platform while eliminating mechanical interference associated with the 3×3 platform. The 6×6 platform analyzed in this paper consists of two rigid bodies connected together by three constant length noncompliant struts and three compliant ties that each consists of a spring in series

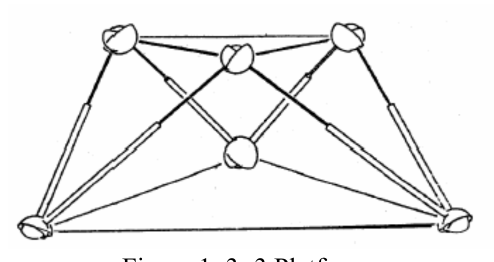

Figure 1: 3×3 Platform

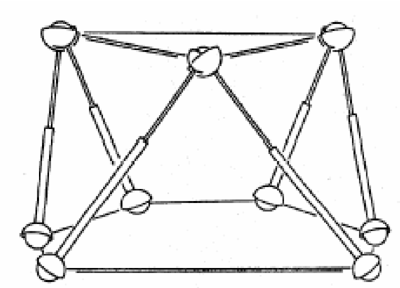

Figure 2: 6×3 Platform

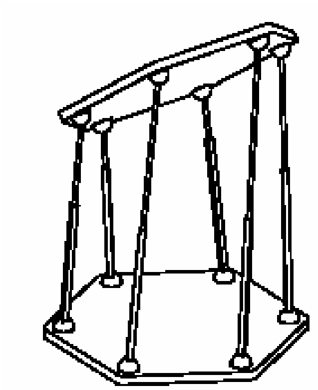

Figure 3: General 6×6 Platform

with a non-compliant tie where the length of the non-compliant tie can be controlled. The legs are connected to the platforms with ball and socket joints. For this analysis, the bottom platform is connected to the ground and the top platform has three degrees of freedom relative to the bottom platform.

MOTIVATION

Prior work (Marshall and Crane, 2004) showed that it is possible to design a parallel platform that incorporates tensegrity principles. Figure 4 shows one version of such a mechanism. Comparing this device with a typical parallel platform such as that in Figure 3, it can be seen that three of the leg connectors have been replaced by three tension members that consist of three compliant springs connected in series with three non-compliant ties. The lengths of the non-compliant ties may be controlled by having an actuator wind the tie about a drum which is primitively illustrated on the base platform in Figure 4. This acts, in effect, to allow the effective free length of the compliant tensile leg connector to be controlled.

Marshall [Marshall, 2003] showed that it is possible to position and orient the top platform to a desired pose at a desired total potential energy level. In that paper it was shown how the lengths of the three leg connectors and the lengths of the three variable length non-compliant ties could be determined to attain the desired pose and potential energy state.

In this paper, the three leg connectors are now replaced by three fixed length struts. The new device, which is similar to that in Figure 4 except that now the three compressive legs are now of fixed length, is now a three degree-of-freedom system where the only variable inputs are the lengths of the three non-compliant ties that are each in series with a spring of known free length and spring constant. Here it will be shown how the lengths of these three non-compliant ties can be determined in

Figure 4: Marshall and Crane's Tensegrity Platform

order to position a point on the top platform at a desired location while the mechanism is at a desired total potential energy state.

The motivation for this work is to determine if there is a means whereby the potential energy in the system can be redistributed in order to reposition the top platform to some desired location in an energy efficient manner. The resulting system has the potential to be a very energy efficient device. In this paper, the lengths of the three struts are fixed, so that for this simple case, the struts will do no work (either positive or negative) as the top platform moves to a new desired point. This paper provides the initial analysis that is required for the platform in order to consider the energy distribution problem in the future.

PROBLEM STATEMENT

This section presents the solution of how to find all possible orientations of the top platform with respect the bottom platform when a point embedded in the top platform is positioned at a desired location. The model used for this analysis is shown in Figure 5. Points B_1, B_2, B_3, and T_1, T_2, T_3 correspond to the centers of the spherical joints at the bottom and top ends of the three constant length noncompliant struts which are numbered 2, 5, and 8 in the figure. Point P is a point that is embedded in the top platform.

Figure 5 shows two coordinate systems that are defined for this problem. The first is attached to the base platform with its origin at point B_1 and X axis through B_2. Point B_3 lies in the

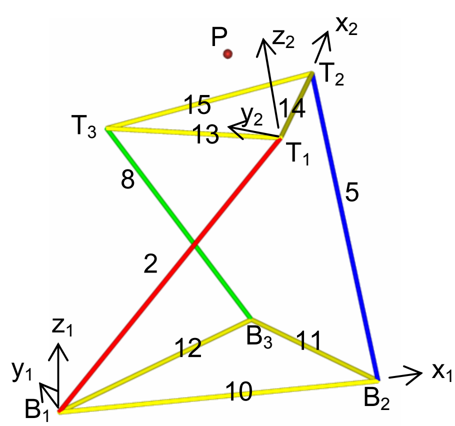

Figure 5: Kinematic Model of 3 dof Tensegrity Platform

XY plane. The second coordinate system is attached to the top platform. The origin is at point T_1, its X axis passes through point T_2 and point T_3 is in the XY plane. The precise problem statement is now presented as follows:

Given

- all dimensions of the top and bottom platforms, i.e. the lengths L_{10}, L_{11}, L_{12}, L_{13}, L_{14}, and L_{15}, where the notation L_i refers to the length of bar i,
- the lengths of the three constant length noncompliant struts, L_2, L_5, and L_8,
- the coordinates of point P in the 2nd (top) coordinate system, which implies that the lengths L_3, L_6, and L_9 shown in Figure 6 are known,
- the coordinates of point P in the 1st (base) coordinate system, which implies that the lengths L_1, L_4, and L_7 shown in Figure 6 are known

Find

- All possible orientations of the top platform

It is important to note that based on the problem statement, the lengths of all fifteen line segments shown in Figure 6 are known. Also it is helpful to visualize the problem as that of having two tetrahedrons, one defined by points B_1, B_2, B_3, and P and the other by points T_1, T_2, T_3, and P, that share the common point P. The problem can be thought of as that of determining all the possible relative orientations of the two tetrahedrons such that the three distance constraints associated with the constant length struts are satisfied.

IDENTIFICATION OF THREE SPHERICAL FOUR-BAR MECHANISMS

This section begins the solution for finding all possible orientations of the top platform with respect to the bottom platform when the point P is positioned as specified.

The analysis begins by defining the three major planes. These planes are special because the position and orientation of these planes are known. These three planes consists of lines (1-7-12), (4-7-11), and (1-4-10). Because these planes all have vertices at known points, the position and orientation can be readily determined.

Several new angles will now be defined, i.e. θ_1, θ_2, θ_3, φ_1,

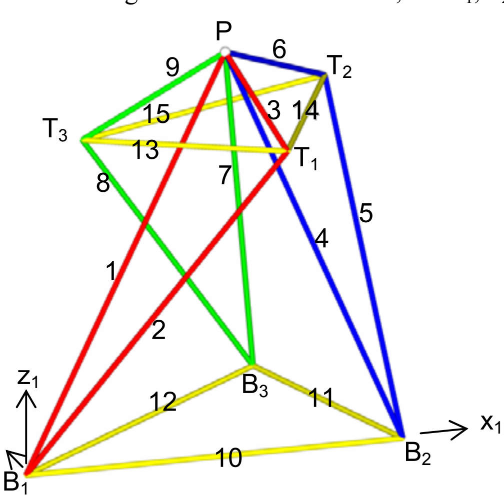

Figure 6: Known Distances Labeled

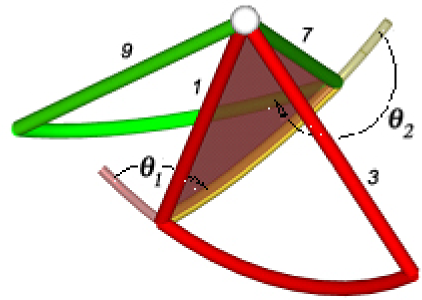

Figure 7: Definition of angles θ_1 and θ_2

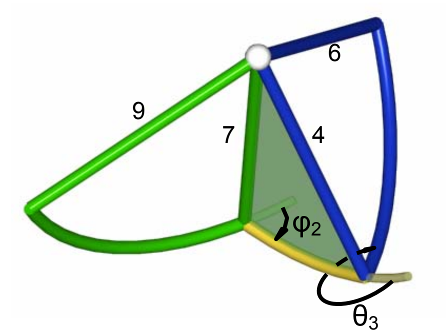

Figure 8: Definition of angles θ_3 and φ_2

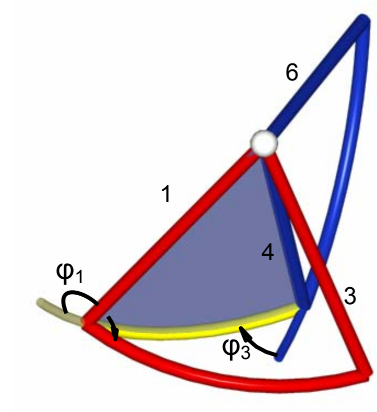

Figure 9: Definition of angles φ_1 and φ_3

φ_2, φ_3, and γ_1, γ_2, γ_3. θ_1 is defined as the angle between planes formed by the lines 1-3 and 1-7 (Figure 7). θ_2 is defined as the angle between planes formed by the lines 9-7 and 1-7 (Figure 7). θ_3 is defined as the angle between planes formed by the lines 4-6 and 4-7 (Figure 8). The definitions of angles φ_1, φ_2, and φ_3 are similar and these are shown in Figures 8 and 9.

The angle γ_2 is defined as the angle between the planes defined by lines 1-7 and 4-7 and is shown in Figure 10. This angle is known since the endpoints of line segments 1, 4, and 7 are known in terms of the base coordinate system. From Figure 10 it can also be seen that

$$\varphi_2 + \gamma_2 + \theta_2 = 0 \,. \tag{1}$$

The angles γ_1 and γ_3 are defined in a manner similar to γ_2 and are also known quantities. Further it can be shown that

$$\varphi_1 + \gamma_1 + \theta_1 = 0 \tag{2}$$

and

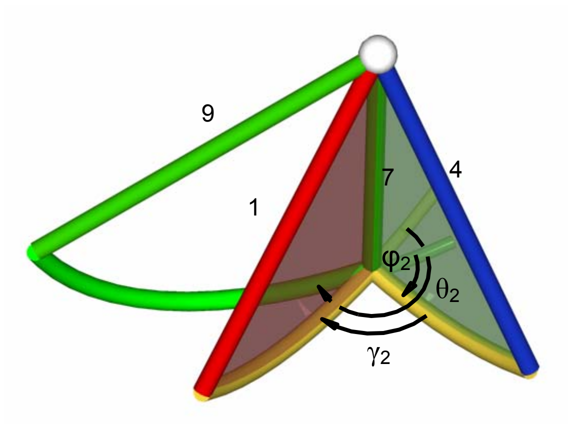

Figure 10: Definition of γ_2

$$\varphi_3 + \gamma_3 + \theta_3 = 0\ . \qquad (3)$$

The analysis can proceed by recognizing that there exist three spherical four bar mechanisms in the model of the device. The "output" angle of one of the four-bar mechanisms relates to the "input" angle of the next spherical four-bar mechanism.

Figure 7 depicts the first spherical four-bar mechanism. It can be seen in Figure 6 that the known length of member 13 maintains a constant angular relationship between members 3 and 9. Similarly Figures 8 and 9 depicts the other two spherical four–bar mechanisms since the angular relationships between 6 and 9 and 3 and 6 are constant and known.

ANALYSIS OF SPHERICAL FOUR-BAR MECHANISM

Figure 11 shows a spherical four-bar mechanism. The links are defined by the angles α_{12}, α_{23}, α_{34}, α_{41} and the relative angles between the links are defined by the angles θ_1 through θ_4. A spherical cosine law for the spherical quadrilateral may be written as (Crane and Duffy, 1998)

$$Z_{41} = c_{23} \qquad (4)$$

where

$$Z_{41} = s_{12}\,(X_4 s_1 + Y_4 c_1) + c_{12}\,Z_4 \qquad (5)$$

and where

$$X_4 = s_{34}\,s_4\ , \qquad (6)$$

$$Y_4 = -(s_{41}c_{34} + c_{41}\,s_{34}\,c_4)\ , \qquad (7)$$

$$Z_4 = c_{41}c_{34} - s_{41}\,s_{34}\,c_4. \qquad (8)$$

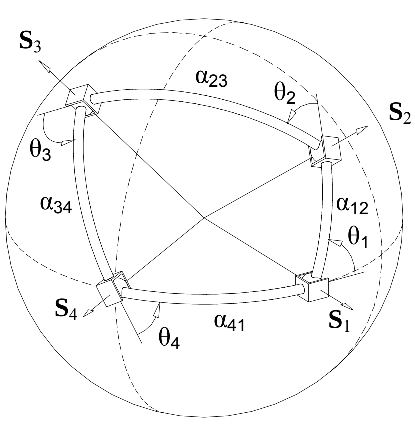

Figure 11: Spherical Four-Bar Mechanism

The terms s_i and c_i in equations (4) through (7) are the sine and cosine of θ_i and the terms s_{ij} and c_{ij} are the sine and cosine of α_{ij}.

Equation (4) can be expanded by substituting (5) through (7) and then rearranged to give

$$(A_1\,c_4 + A_2)\,c_1 + A_3\,s_1\,s_4 + A_4\,c_4 + A_5 = 0 \qquad (9)$$

where

$$A_1 = -s_{12}\,c_{41}\,s_{34}\ , \qquad (10)$$

$$A_2 = -s_{12}\,s_{41}\,c_{34}\ , \qquad (11)$$

$$A_3 = s_{12}\,s_{34}\ , \qquad (12)$$

$$A_4 = -c_{12}\,c_{41}\,s_{34}\ , \qquad (13)$$

$$A_5 = c_{12}\,c_{41}\,c_{34} - c_{23}\ . \qquad (14)$$

DETERMINATION OF PLATFORM ORIENTATION

Equation (4) may be applied to the three spherical mechanisms to generate three equations, one that relates θ_1 and θ_2, one that relates φ_2 and θ_3, and one that relates φ_1 and φ_3. Equations (1) through (3) may then be used to replace φ_1, φ_2, and φ_3 in terms of θ_1, θ_2, and θ_3. The end result will be three equations in the three unknowns θ_1, θ_2, and θ_3.

Table 1 shows the variable substitution scheme that is used to generate the three equations in the variables θ_i and φ_i, i=1..3 from the generic spherical quadrilateral equation, (16).

Table 1: Variable Substitution for Spherical Mechanisms

Generic Quad.	θ_4	θ_1	$\alpha_{4\text{-}1}$	$\alpha_{1\text{-}2}$	$\alpha_{2\text{-}3}$	$\alpha_{3\text{-}4}$
1st Quad.	θ_1	θ_2	$\alpha_{1\text{-}7}$	$\alpha_{7\text{-}9}$	$\alpha_{9\text{-}3}$	$\alpha_{3\text{-}1}$
2nd Quad.	φ_2	θ_3	$\alpha_{7\text{-}4}$	$\alpha_{4\text{-}6}$	$\alpha_{6\text{-}9}$	$\alpha_{9\text{-}7}$
3rd Quad.	φ_3	φ_1	$\alpha_{4\text{-}1}$	$\alpha_{1\text{-}3}$	$\alpha_{3\text{-}6}$	$\alpha_{6\text{-}4}$

Applying the substitutions in Table 1 to equation (9) yields the following three equations:

$$(G_1\,c_1 + G_2)\,c_2 + G_3\,s_2\,s_1 + G_4\,c_1 + G_5 = 0 \qquad (15)$$

$$(H_1\cos\varphi_2 + H_2)\,c_3 + H_3\,s_3\sin\varphi_2 + H_4\cos\varphi_2 + H_5 = 0 \qquad (16)$$

$$(I_1\cos\varphi_3 + I_2)\cos\varphi_1 + I_3\sin\varphi_1\sin\varphi_3 + I_4\cos\varphi_3 + I_5 = 0 \qquad (17)$$

where the coefficients G_1 through I_5 are quantities that are expressed in terms of the constant mechanism parameters. Here the notation s_i and c_i are used to represent the sine and cosine of θ_i.

Equations (1) through (3) may now be used to substitute for the angles φ_i in terms of θ_i in (15) through (17). Performing this step and replacing the sines and cosines of θ_i by the trigonometric identities

$$\sin\theta_i = \frac{2\,x_i}{1+x_i^2}\ , \quad \cos\theta_i = \frac{1-x_i^2}{1+x_i^2} \qquad (18)$$

where $x_i = \tan(\theta_i)/2$ and regrouping gives

$$(A_9x_1^2 + A_8x_1 + A_7)\,x_2^2 + (A_6x_1^2 + A_5x_1 + A_4)\,x_2 + (A_3x_1^2 + A_2x_1 + A_1) = 0\ , \qquad (19)$$

$$(B_9x_3^2 + B_8x_3 + B_7)\,x_2^2 + (B_6x_3^2 + B_5x_3 + B_4)\,x_2 + (B_3x_3^2 + B_2x_3 + B_1) = 0\ , \qquad (20)$$

$$(D_9x_1^2 + D_8x_1 + D_7)\, x_3^2 + (D_6x_1^2 + D_5x_1 + D_4)\, x_3 + (D_3x_1^2 + D_2x_1 + D_1) = 0 \qquad (21)$$

where the coefficients A_9 through D_1 are known quantities.

The problem at hand is to determine all sets of x_1, x_2, and x_3 that will simultaneously satisfy the set of equations (19), (20), and (21). Values for θ_i may then be obtained from each value of x_i as $\theta_i = 2\tan^{-1}(x_i)$.

The solution procedure will begin by using Bezout's method to eliminate x_2 from equations (19) and (20), resulting in a new equation in x_1 and x_3. Sylvester's method will then be used with this new equation, together with (21), to eliminate x_3 to obtain a single polynomial in x_1.

To start the solution process, let

$$\begin{aligned} L_1 &= A_9x_1^2 + A_8x_1 + A_7\,, \\ M_1 &= A_6x_1^2 + A_5x_1 + A_4\,, \\ N_1 &= A_3x_1^2 + A_2x_1 + A_1 \end{aligned} \qquad (22)$$

and

$$\begin{aligned} L_2 &= B_9x_1^2 + B_8x_1 + B_7\,, \\ M_2 &= B_6x_1^2 + B_5x_1 + B_4\,, \\ N_2 &= B_3x_1^2 + B_2x_1 + B_1\,. \end{aligned} \qquad (23)$$

Equations (19) and (20) may now be written as

$$L_1\, x_2^2 + M_1\, x_2 + N_1 = 0\,, \qquad (24)$$

$$L_2\, x_2^2 + M_2\, x_2 + N_2 = 0\,. \qquad (25)$$

The condition that (24) and (25) have a common root for x_2 is

$$\begin{bmatrix} L_1 & M_1 \\ L_2 & M_2 \end{bmatrix}\begin{bmatrix} M_1 & N_1 \\ M_2 & N_2 \end{bmatrix} - \begin{bmatrix} L_1 & N_1 \\ L_2 & N_2 \end{bmatrix}^2 = 0\,. \qquad (26)$$

Expanding (26) and collecting terms gives

$$V_4\, x_3^4 + V_3\, x_3^3 + V_2\, x_3^2 + V_1\, x_3 + V_0 = 0 \qquad (27)$$

where

$$\begin{aligned} V_4 &= V_{44}\, x_1^4 + V_{43}\, x_1^3 + V_{42}\, x_1^2 + V_{41}\, x_1 + V_{40}\,, \\ V_3 &= V_{34}\, x_1^4 + V_{33}\, x_1^3 + V_{32}\, x_1^2 + V_{31}\, x_1 + V_{30}\,, \\ V_2 &= V_{24}\, x_1^4 + V_{23}\, x_1^3 + V_{22}\, x_1^2 + V_{21}\, x_1 + V_{20}\,, \\ V_1 &= V_{14}\, x_1^4 + V_{13}\, x_1^3 + V_{12}\, x_1^2 + V_{11}\, x_1 + V_{10}\,, \\ V_0 &= V_{04}\, x_1^4 + V_{03}\, x_1^3 + V_{02}\, x_1^2 + V_{01}\, x_1 + V_{00} \end{aligned} \qquad (28)$$

and the coefficients V_{ij} are known quantitites.

The terms W_2, W_1, and W_0 are now defined as

$$\begin{aligned} W_2 &= D_9x_1^2 + D_8x_1 + D_7\,, \\ W_1 &= D_6x_1^2 + D_5x_1 + D_4\,, \\ W_0 &= D_3x_1^2 + D_2x_1 + D_1\,. \end{aligned} \qquad (29)$$

Substituting these expressions into (21) yields

$$W_2\, x_3^2 + W_1\, x_3 + W_0 = 0\,. \qquad (30)$$

Sylvester's elimination method is then used to eliminate x_3 from the pair of equations (27) and (30). Multiplying (27) by x_3 and (30) by x_3, x_3^2, and x_3^3 results in six equations that can be written in matrix format as

$$\begin{bmatrix} 0 & 0 & 0 & W_2 & W_1 & W_0 \\ 0 & 0 & W_2 & W_1 & W_0 & 0 \\ 0 & W_2 & W_1 & W_0 & 0 & 0 \\ W_2 & W_1 & W_0 & 0 & 0 & 0 \\ 0 & V_4 & V_3 & V_2 & V_1 & V_0 \\ V_4 & V_3 & V_2 & V_1 & V_0 & 0 \end{bmatrix}\begin{bmatrix} x_3^5 \\ x_3^4 \\ x_3^3 \\ x_3^2 \\ x_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (31)$$

This set of six 'homogeneous' equations will have a solution only if the equations are linearly dependent and thus the determinant of the coefficient matrix must equal zero. Thus

$$\begin{vmatrix} 0 & 0 & 0 & W_2 & W_1 & W_0 \\ 0 & 0 & W_2 & W_1 & W_0 & 0 \\ 0 & W_2 & W_1 & W_0 & 0 & 0 \\ W_2 & W_1 & W_0 & 0 & 0 & 0 \\ 0 & V_4 & V_3 & V_2 & V_1 & V_0 \\ V_4 & V_3 & V_2 & V_1 & V_0 & 0 \end{vmatrix} = 0\,. \qquad (32)$$

Since the terms V_i and W_i are polynomials in x_1, expanding this determinant yields a 16[th] degree polynomial in x_1. Each value of x_1 obtained by the polynomial can be substituted back into (31) to obtain corresponding values for x_3. Once the corresponding values of x_3 have been determined for each value of x_1, corresponding values for x_2 may be obtained from (24) and (25) as

$$x_2 = \frac{-\begin{vmatrix} M_1 & N_1 \\ M_2 & N_2 \end{vmatrix}}{\begin{vmatrix} L_1 & N_1 \\ L_2 & N_2 \end{vmatrix}}\,, \qquad (33)$$

or

$$x_2 = \frac{-\begin{vmatrix} L_1 & N_1 \\ L_2 & N_2 \end{vmatrix}}{\begin{vmatrix} L_1 & M_1 \\ L_2 & M_2 \end{vmatrix}}\,. \qquad (34)$$

Lastly, values for the solution sets $\{\theta_1, \theta_2, \theta_3\}$ are obtained from the corresponding values of x_i as $\theta_i = 2\tan^{-1}(x_i)$.

The coordinates of the points T_1, T_2, and T_3 may be obtained in terms of the base coordinate system for each solution set $\{\theta_1, \theta_2, \theta_3\}$. The procedure is not presented here, but is straightforward. Knowledge of the coordinates of these three points allows for the determination of the 4×4 transformation ${}^1_2\mathbf{T}$ that describes the relative position and orientation of the coordinate system attached to the top platform to that attached to the base.

NUMERICAL EXAMPLE

A numerical example is presented to verify the results. The following information was used as input for the numerical case:

Given:

- ${}^1\mathbf{P}$ = (6.2486, 10, -3.4732) ; point P in 1st coord. system
- ${}^2\mathbf{P}$ = (1.7590, 2, -1.9251) ; point P in 2nd coord. system
- ${}^1\mathbf{P}_{B1}$ = (0, 0, 0) ; point B_1 in 1[st] coord. system
- ${}^1\mathbf{P}_{B2}$ = (10, 0, 0) ; point B_2 in 1[st] coord. system
- ${}^1\mathbf{P}_{B3}$ = (6.75, -5.6292, 0) ; point B_3 in 1[st] coord. system

- ${}^{2}\mathbf{P}_{T1} = (0, 0, 0)$; point T_1 in 2nd coord. system
- ${}^{2}\mathbf{P}_{T2} = (4.9500, 0, 0)$; point T_2 in 2nd coord. system
- ${}^{2}\mathbf{P}_{T3} = (0.2658, -5.5675, 0)$; point T_3 in 2nd coord. system
- $L_2 = 11.035$; length of strut 2
- $L_5 = 9.970$; length of strut 3
- $L_8 = 9.455$; length of strut 4

The units of length are immaterial to this problem as long as the same unit is used throughout the analysis.

Table 2 shows the calculated values for the fifteen segments shown in Figure 6.

Table 2: Segment Lengths

Segment	Length
1	12.292
2	11.035
3	3.286
4	11.231
5	9.970
6	4.232
7	10.242
8	9.455
9	4.415
10	10.000
11	6.500
12	8.789
13	5.573
14	4.954
15	7.278

The 16th degree polynomial in x_1 was obtained from (32). For this case, four of the roots of this polynomial were real. The four real cases, and the corresponding values for x_2 and x_3 are shown in Table 3.

Table 3: Results of Analysis

	Case 1	Case 2	Case 3	Case 4
x_1	1.2077	-2.307	4.1559	4.6202
x_2	-3.716	0.3637	-1.057	-2.153
x_3	-1.630	1.0044	0.5439	0.1679

From this information, the angles θ_1, θ_2, and θ_3 and then the coordinates of points T_1, T_2, and T_3 measured in the base coordinate system were determined. Figures 12 through 15 show the device in the four attainable configurations.

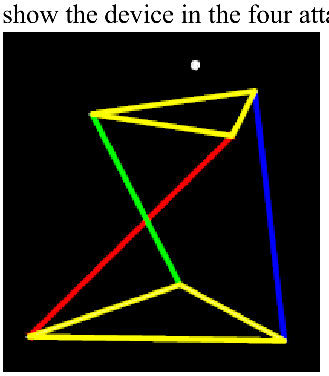

Figure 12: Case 1

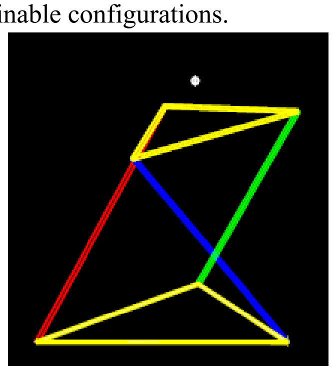

Figure 13: Case 2

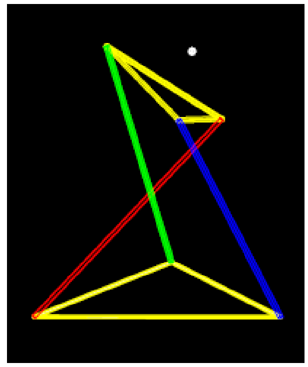

Figure 14: Case 3

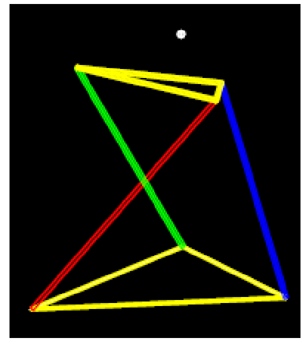

Figure 15: Case 4

CONCLUSION

This study successfully analyzed a three degree of freedom tensegrity platform that incorporates the special 6×6 platform geometry. This analysis shows that if a coordinate system attached to the bottom platform is specified, and a predetermined location of a point attached to the top platform is also specified, then the orientation of the top platform with respect to the coordinate system of the bottom platform can be determined. This analysis focused on a closed form algebraic solution of the platform. The analysis proceeded by defining three dependent spherical quadrilaterals.

Although iterative methods could have also given the orientation of the top platform, these methods would leave out critical information. By using iterative procedures, one would never know how many orientations are possible of the top platform for a given position P. For example, a numerical case shown in this paper identified four possible orientations of the top platform for the given point P. Using an iterative procedure, the solution of the orientation of the top platform could possibly converge at any one of the four orientations. The solution of the iterative method is almost entirely based on the initial values given to the method. Given this analysis is much more rigorous than iterative methods, the information obtained about the system makes up for the additional effort. It was learned that there are sixteen possible total orientations of the top platform.

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